Naturally occurring hydrocarbon systems found in petroleum reservoirs are mixtures of organic compounds which exhibit multiphase behavior over wide ranges of pressures and temperatures. These hydrocarbon accumulations may occur in the gaseous state, the liquid state, the solid state, or in various combinations of gas, liquid, and solid.

These differences in phase behavior, coupled with the physical properties of reservoir rock that determine the relative ease with which gas and liquid are transmitted or retained, result in many diverse types of hydrocarbon reservoirs with complex behaviors. Frequently, petroleum engineers have the task to study the behavior and characteristics of a petroleum reservoir and to determine the course of future development and production that would maximize the profit.

The objective of this chapter is to review the basic principles of reservoir fluid phase behavior and illustrate the use of phase diagrams in classifying types of reservoirs and the native hydrocarbon systems.

CLASSIFICATION OF RESERVOIRS AND RESERVOIR FLUIDS

Petroleum reservoirs are broadly classified as oil or gas reservoirs. These broad classifications are further subdivided depending on:
• The composition of the reservoir hydrocarbon mixture
• Initial reservoir pressure and temperature
• Pressure and temperature of the surface production

The conditions under which these phases exist are a matter of considerable practical importance. The experimental or the mathematical determinations of these conditions are conveniently expressed in different types of diagrams commonly called phase diagrams. One such diagram is called the pressure-temperature diagram.

**Pressure-Temperature Diagram**

Figure 1-1 shows a typical pressure-temperature diagram of a multicomponent system with a specific overall composition. Although a different hydrocarbon system would have a different phase diagram, the general configuration is similar.

![Figure 1-1. Typical p-T diagram for a multicomponent system.](image-url)
These multicomponent pressure-temperature diagrams are essentially used to:

- Classify reservoirs
- Classify the naturally occurring hydrocarbon systems
- Describe the phase behavior of the reservoir fluid

To fully understand the significance of the pressure-temperature diagrams, it is necessary to identify and define the following key points on these diagrams:

- **Cricondentherm** ($T_{ct}$)—The Cricondentherm is defined as the maximum temperature above which liquid cannot be formed regardless of pressure (point E). The corresponding pressure is termed the Cricondentherm pressure $p_{ct}$.

- **Cricondenbar** ($p_{cb}$)—The Cricondenbar is the maximum pressure above which no gas can be formed regardless of temperature (point D). The corresponding temperature is called the Cricondenbar temperature $T_{cb}$.

- **Critical point**—The critical point for a multicomponent mixture is referred to as the state of pressure and temperature at which all intensive properties of the gas and liquid phases are equal (point C). At the critical point, the corresponding pressure and temperature are called the critical pressure $p_c$ and critical temperature $T_c$ of the mixture.

- **Phase envelope (two-phase region)**—The region enclosed by the bubble-point curve and the dew-point curve (line BCA), wherein gas and liquid coexist in equilibrium, is identified as the phase envelope of the hydrocarbon system.

- **Quality lines**—The dashed lines within the phase diagram are called quality lines. They describe the pressure and temperature conditions for equal volumes of liquids. Note that the quality lines converge at the critical point (point C).

- **Bubble-point curve**—The bubble-point curve (line BC) is defined as the line separating the liquid-phase region from the two-phase region.

- **Dew-point curve**—The dew-point curve (line AC) is defined as the line separating the vapor-phase region from the two-phase region.

In general, reservoirs are conveniently classified on the basis of the location of the point representing the initial reservoir pressure $p_i$ and temperature $T$ with respect to the pressure-temperature diagram of the reservoir fluid. Accordingly, reservoirs can be classified into basically two types. These are:
• **Oil reservoirs**—If the reservoir temperature \( T \) is less than the critical temperature \( T_c \) of the reservoir fluid, the reservoir is classified as an oil reservoir.

• **Gas reservoirs**—If the reservoir temperature is greater than the critical temperature of the hydrocarbon fluid, the reservoir is considered a gas reservoir.

**Oil Reservoirs**

Depending upon initial reservoir pressure \( p_i \), oil reservoirs can be sub-classified into the following categories:

1. **Undersaturated oil reservoir.** If the initial reservoir pressure \( p_i \) (as represented by point 1 on Figure 1-1), is greater than the bubble-point pressure \( p_b \) of the reservoir fluid, the reservoir is labeled an undersaturated oil reservoir.

2. **Saturated oil reservoir.** When the initial reservoir pressure is equal to the bubble-point pressure of the reservoir fluid, as shown on Figure 1-1 by point 2, the reservoir is called a saturated oil reservoir.

3. **Gas-cap reservoir.** If the initial reservoir pressure is below the bubble-point pressure of the reservoir fluid, as indicated by point 3 on Figure 1-1, the reservoir is termed a gas-cap or two-phase reservoir, in which the gas or vapor phase is underlain by an oil phase. The appropriate quality line gives the ratio of the gas-cap volume to reservoir oil volume.

Crude oils cover a wide range in physical properties and chemical compositions, and it is often important to be able to group them into broad categories of related oils. In general, crude oils are commonly classified into the following types:

• Ordinary black oil
• Low-shrinkage crude oil
• High-shrinkage (volatile) crude oil
• Near-critical crude oil

The above classifications are essentially based upon the properties exhibited by the crude oil, including physical properties, composition, gas-oil ratio, appearance, and pressure-temperature phase diagrams.

1. **Ordinary black oil.** A typical pressure-temperature phase diagram for ordinary black oil is shown in Figure 1-2. It should be noted that quality lines which are approximately equally spaced characterize this
black oil phase diagram. Following the pressure reduction path as indicated by the vertical line EF on Figure 1-2, the liquid shrinkage curve, as shown in Figure 1-3, is prepared by plotting the liquid volume percent as a function of pressure. The liquid shrinkage curve approximates a straight line except at very low pressures. When produced, ordinary black oils usually yield gas-oil ratios between 200–700 scf/STB and oil gravities of 15 to 40 API. The stock tank oil is usually brown to dark green in color.

2. **Low-shrinkage oil.** A typical pressure-temperature phase diagram for low-shrinkage oil is shown in Figure 1-4. The diagram is characterized by quality lines that are closely spaced near the dew-point curve. The liquid-shrinkage curve, as given in Figure 1-5, shows the shrinkage characteristics of this category of crude oils. The other associated properties of this type of crude oil are:

- Oil formation volume factor less than 1.2 bbl/STB
- Gas-oil ratio less than 200 scf/STB
- Oil gravity less than 35° API
- Black or deeply colored

**Figure 1-2.** A typical p-T diagram for an ordinary black oil.
3. **Volatile crude oil.** The phase diagram for a volatile (high-shrinkage) crude oil is given in Figure 1-6. Note that the quality lines are close
together near the bubble-point and are more widely spaced at lower pressures. This type of crude oil is commonly characterized by a high liquid shrinkage immediately below the bubble-point as shown in Figure 1-7. The other characteristic properties of this oil include:
Another characteristic of volatile oil reservoirs is that the API gravity of the stock-tank liquid will increase in the later life of the reservoirs.

4. **Near-critical crude oil.** If the reservoir temperature $T$ is near the critical temperature $T_c$ of the hydrocarbon system, as shown in Figure 1-8, the hydrocarbon mixture is identified as a near-critical crude oil. Because all the quality lines converge at the critical point, an isothermal pressure drop (as shown by the vertical line EF in Figure 1-8) may shrink the crude oil from 100% of the hydrocarbon pore volume at the bubble-point to 55% or less at a pressure 10 to 50 psi below the bubble-point. The shrinkage characteristic behavior of the near-critical crude oil is shown in Figure 1-9. The near-critical crude oil is characterized by a high GOR in excess of 3,000 scf/STB with an oil formation volume factor of 2.0 bbl/STB or higher. The compositions of near-critical oils are usually characterized by 12.5 to 20 mol% heptanes-plus, 35% or more of ethane through hexanes, and the remainder methane.
Figure 1-8. A schematic phase diagram for the near-critical crude oil.

Figure 1-9. A typical liquid-shrinkage curve for the near-critical crude oil.
Figure 1-10 compares the characteristic shape of the liquid-shrinkage curve for each crude oil type.

**Gas Reservoirs**

In general, if the reservoir temperature is above the critical temperature of the hydrocarbon system, the reservoir is classified as a natural gas reservoir. On the basis of their phase diagrams and the prevailing reservoir conditions, natural gases can be classified into four categories:

- Retrograde gas-condensate
- Near-critical gas-condensate
- Wet gas
- Dry gas

**Retrograde gas-condensate reservoir.** If the reservoir temperature $T$ lies between the critical temperature $T_c$ and cricondentherm $T_{ct}$ of the reservoir fluid, the reservoir is classified as a retrograde gas-condensate reservoir. This category of gas reservoir is a unique type of hydrocarbon accumulation in that the special thermodynamic behavior of the reservoir fluid is the controlling factor in the development and the depletion process of the reservoir. When the pressure is decreased on these mix-
tures, instead of expanding (if a gas) or vaporizing (if a liquid) as might be expected, they vaporize instead of condensing.

Consider that the initial condition of a retrograde gas reservoir is represented by point 1 on the pressure-temperature phase diagram of Figure 1-11. Because the reservoir pressure is above the upper dew-point pressure, the hydrocarbon system exists as a single phase (i.e., vapor phase) in the reservoir. As the reservoir pressure declines isothermally during production from the initial pressure (point 1) to the upper dew-point pressure (point 2), the attraction between the molecules of the light and heavy components causes them to move further apart further apart. As this occurs, attraction between the heavy component molecules becomes more effective; thus, liquid begins to condense.

This retrograde condensation process continues with decreasing pressure until the liquid dropout reaches its maximum at point 3. Further reduction in pressure permits the heavy molecules to commence the normal vaporization process. This is the process whereby fewer gas molecules strike the liquid surface and causes more molecules to leave than

![Figure 1-11. A typical phase diagram of a retrograde system.](image-url)
enter the liquid phase. The vaporization process continues until the reservoir pressure reaches the lower dew-point pressure. This means that all the liquid that formed must vaporize because the system is essentially all vapors at the lower dew point.

Figure 1-12 shows a typical liquid shrinkage volume curve for a condensate system. The curve is commonly called the **liquid dropout curve**. In most gas-condensate reservoirs, the condensed liquid volume seldom exceeds more than 15%–19% of the pore volume. This liquid saturation is not large enough to allow any liquid flow. It should be recognized, however, that around the wellbore where the pressure drop is high, enough liquid dropout might accumulate to give two-phase flow of gas and retrograde liquid.

The associated physical characteristics of this category are:

- Gas-oil ratios between 8,000 to 70,000 scf/STB. Generally, the gas-oil ratio for a condensate system increases with time due to the liquid dropout and the loss of heavy components in the liquid.
- Condensate gravity above 50° API
- Stock-tank liquid is usually water-white or slightly colored.

There is a fairly sharp dividing line between oils and condensates from a compositional standpoint. Reservoir fluids that contain heptanes and are heavier in concentrations of more than 12.5 mol% are almost always in the liquid phase in the reservoir. Oils have been observed with hep-
tanes and heavier concentrations as low as 10% and condensates as high as 15.5%. These cases are rare, however, and usually have very high tank liquid gravities.

**Near-critical gas-condensate reservoir.** If the reservoir temperature is near the critical temperature, as shown in Figure 1-13, the hydrocarbon mixture is classified as a near-critical gas-condensate. The volumetric behavior of this category of natural gas is described through the isothermal pressure declines as shown by the vertical line 1-3 in Figure 1-13 and also by the corresponding liquid dropout curve of Figure 1-14. Because all the quality lines converge at the critical point, a rapid liquid buildup will immediately occur below the dew point (Figure 1-14) as the pressure is reduced to point 2.

![Figure 1-13](image.png)

**Figure 1-13.** A typical phase diagram for a near-critical gas condensate reservoir.
This behavior can be justified by the fact that several quality lines are crossed very rapidly by the isothermal reduction in pressure. At the point where the liquid ceases to build up and begins to shrink again, the reservoir goes from the retrograde region to a normal vaporization region.

**Wet-gas reservoir.** A typical phase diagram of a wet gas is shown in Figure 1-15, where reservoir temperature is above the cricondentherm of the hydrocarbon mixture. Because the reservoir temperature exceeds the cricondentherm of the hydrocarbon system, the reservoir fluid will always remain in the vapor phase region as the reservoir is depleted isothermally, along the vertical line A-B.

As the produced gas flows to the surface, however, the pressure and temperature of the gas will decline. If the gas enters the two-phase region, a liquid phase will condense out of the gas and be produced from the surface separators. This is caused by a sufficient decrease in the kinetic energy of heavy molecules with temperature drop and their subsequent change to liquid through the attractive forces between molecules.

Wet-gas reservoirs are characterized by the following properties:

- Gas oil ratios between 60,000 to 100,000 scf/STB
- Stock-tank oil gravity above 60° API
- Liquid is water-white in color
- Separator conditions, i.e., separator pressure and temperature, lie within the two-phase region

![Figure 1-14. Liquid-shrinkage curve for a near-critical gas-condensate system.](image-url)
Dry-gas reservoir. The hydrocarbon mixture exists as a gas both in the reservoir and in the surface facilities. The only liquid associated with the gas from a dry-gas reservoir is water. A phase diagram of a dry-gas reservoir is given in Figure 1-16. Usually a system having a gas-oil ratio greater than 100,000 scf/STB is considered to be a dry gas.

Kinetic energy of the mixture is so high and attraction between molecules so small that none of them coalesce to a liquid at stock-tank conditions of temperature and pressure.

It should be pointed out that the classification of hydrocarbon fluids might be also characterized by the initial composition of the system. McCain (1994) suggested that the heavy components in the hydrocarbon mixtures have the strongest effect on fluid characteristics. The ternary diagram, as shown in Figure 1-17, with equilateral triangles can be conveniently used to roughly define the compositional boundaries that separate different types of hydrocarbon systems.
From the foregoing discussion, it can be observed that hydrocarbon mixtures may exist in either the gaseous or liquid state, depending on the reservoir and operating conditions to which they are subjected. The qualitative concepts presented may be of aid in developing quantitative analyses. Empirical equations of state are commonly used as a quantitative tool in describing and classifying the hydrocarbon system. These equations of state require:

- Detailed compositional analyses of the hydrocarbon system
- Complete descriptions of the physical and critical properties of the mixture individual components

Many characteristic properties of these individual components (in other words, pure substances) have been measured and compiled over the years. These properties provide vital information for calculating the thermodynamic properties of pure components, as well as their mixtures. The most important of these properties are:

Figure 1-16. Phase diagram for a dry gas. (After Clark, N.J. Elements of Petroleum Reservoirs, SPE, 1969.)
Table 1-2 documents the above-listed properties for a number of hydrocarbon and nonhydrocarbon components.

Katz and Firoozabadi (1978) presented a generalized set of physical properties for the petroleum fractions $C_6$ through $C_{45}$. The tabulated properties include the average boiling point, specific gravity, and molecular weight. The authors’ proposed a set of tabulated properties that were generated by analyzing the physical properties of 26 condensates and crude oil systems. These generalized properties are given in Table 1-1.
### Table 1-1

**Generalized Physical Properties**

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<th>Group</th>
<th>$T_b$ ($^\circ$R)</th>
<th>$\gamma$</th>
<th>$K$</th>
<th>$M$</th>
<th>$T_c$ ($^\circ$R)</th>
<th>$P_c$ (psia)</th>
<th>$\omega$</th>
<th>$V_t$ ($ft^3/lb$)</th>
<th>Group</th>
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# Table 1-2

Physical Properties for Pure Components

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Table 1-2 (continued)

Physical Constants

*See the Table of Notes and References.

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<th>Temperature coefficient of density, 1°F</th>
<th>Compressibility factor, a</th>
<th>Acoustic factor, α</th>
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<th>Compressibility factor of real gas, K</th>
<th>Relative density</th>
<th>c_p,gas (Btu/lbm-°F)</th>
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Ahmed (1985) correlated Katz-Firoozabadi-tabulated physical properties with the number of carbon atoms of the fraction by using a regression model. The generalized equation has the following form:

\[ \theta = a_1 + a_2 \ n + a_3 \ n^2 + a_4 \ n^3 + \left( a_5 / n \right) \]  

(1-1)

where \( \theta \) = any physical property  
\( n \) = number of carbon atoms, i.e., 6, 7 . . . . , 45  
\( a_1\)–\( a_5 \) = coefficients of the equation and are given in Table 1-3

### Table 1-3  
Coefficients of Equation 1-1

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<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
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<td>1.7547264 \times 10^{-7}</td>
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### Undefined Petroleum Fractions

Nearly all naturally occurring hydrocarbon systems contain a quantity of heavy fractions that are not well defined and are not mixtures of discretely identified components. These heavy fractions are often lumped together and identified as the plus fraction, e.g., \( C_7^{+} \) fraction.

A proper description of the physical properties of the plus fractions and other undefined petroleum fractions in hydrocarbon mixtures is essential in performing reliable phase behavior calculations and compositional modeling studies. Frequently, a distillation analysis or a chromatographic analysis is available for this undefined fraction. Other physical properties, such as molecular weight and specific gravity, may also be measured for the entire fraction or for various cuts of it.

To use any of the thermodynamic property-prediction models, e.g., equation of state, to predict the phase and volumetric behavior of complex hydrocarbon mixtures, one must be able to provide the acentric factor, along with the critical temperature and critical pressure, for both the
defined and undefined (heavy) fractions in the mixture. The problem of how to adequately characterize these undefined plus fractions in terms of their critical properties and acentric factors has been long recognized in the petroleum industry. Whitson (1984) presented an excellent documentation on the influence of various heptanes-plus \((C_7+)\) characterization schemes on predicting the volumetric behavior of hydrocarbon mixtures by equations-of-state.

Riazi and Daubert (1987) developed a simple two-parameter equation for predicting the physical properties of pure compounds and undefined hydrocarbon mixtures. The proposed generalized empirical equation is based on the use of the molecular weight \(M\) and specific gravity \(g\) of the undefined petroleum fraction as the correlating parameters. Their mathematical expression has the following form:

\[
\theta = a (M)^b g^c \exp \left[ d (M) + e \gamma + f (M) \gamma \right]
\]  

(1-2)

where

- \(\theta\) = any physical property
- \(a–f\) = constants for each property as given in Table 1-4
- \(\gamma\) = specific gravity of the fraction
- \(M\) = molecular weight
- \(T_c\) = critical temperature, °R
- \(P_c\) = critical pressure, psia (Table 1-4)
- \(T_b\) = boiling point temperature, °R
- \(V_c\) = critical volume, ft³/lb

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_c), °R</td>
<td>544.4</td>
<td>0.2998</td>
<td>1.0555</td>
<td>-1.3478 \times 10^{-4}</td>
<td>-0.61641</td>
<td>0.0</td>
</tr>
<tr>
<td>(P_c), psia</td>
<td>4.5203 \times 10^6</td>
<td>-0.8063</td>
<td>1.6015</td>
<td>-1.8078 \times 10^{-3}</td>
<td>-0.3084</td>
<td>0.0</td>
</tr>
<tr>
<td>(V_c), ft³/lb</td>
<td>1.206 \times 10^{-3}</td>
<td>0.20378</td>
<td>-1.3036</td>
<td>-2.657 \times 10^{-3}</td>
<td>0.5287</td>
<td>2.6012 \times 10^{-3}</td>
</tr>
<tr>
<td>(T_b), °R</td>
<td>6.77857</td>
<td>0.401673</td>
<td>-1.58262</td>
<td>3.77409 \times 10^{-3}</td>
<td>2.984036</td>
<td>-4.25288 \times 10^{-3}</td>
</tr>
</tbody>
</table>

Edmister (1958) proposed a correlation for estimating the acentric factor \(T\) of pure fluids and petroleum fractions. The equation, widely used in the petroleum industry, requires boiling point, critical temperature, and critical pressure. The proposed expression is given by the following relationship:
where \(\omega = \frac{3 [\log (p_c / 14.70)]}{7 [(T_c / T_b) - 1]} - 1\) (1-3)

where \(T = \) acentric factor
\(p_c = \) critical pressure, psia
\(T_c = \) critical temperature, °R
\(T_b = \) normal boiling point, °R

If the acentric factor is available from another correlation, the Edmistrer equation can be rearranged to solve for any of the three other properties (providing the other two are known).

The critical compressibility factor is another property that is often used in thermodynamic-property prediction models. It is defined as the component compressibility factor calculated at its critical point. This property can be conveniently computed by the real gas equation-of-state at the critical point, or

\[ z_c = \frac{p_c V_c M}{R T_c} \] (1-4)

where \(R = \) universal gas constant, 10.73 psia-ft\(^3\)/lb-mol. °R
\(V_c = \) critical volume, ft\(^3\)/lb
\(M = \) molecular weight

The accuracy of Equation 1-4 depends on the accuracy of the values of \(p_c, T_c,\) and \(V_c\) used in evaluating the critical compressibility factor. Table 1-5 presents a summary of the critical compressibility estimation methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Year</th>
<th>(z_c)</th>
<th>Equation No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haugen</td>
<td>1959</td>
<td>(z_c = 1/(1.28 \omega + 3.41))</td>
<td>1-5</td>
</tr>
<tr>
<td>Reid, Prausnitz, and</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sherwood</td>
<td>1977</td>
<td>(z_c = 0.291 - 0.080 \omega)</td>
<td>1-6</td>
</tr>
<tr>
<td>Salerno, et al.</td>
<td>1985</td>
<td>(z_c = 0.291 - 0.080 \omega - 0.016 \omega^2)</td>
<td>1-7</td>
</tr>
<tr>
<td>Nath</td>
<td>1985</td>
<td>(z_c = 0.2918 - 0.0928)</td>
<td>1-8</td>
</tr>
</tbody>
</table>
Example 1-1

Estimate the critical properties and the acentric factor of the heptanes-plus fraction, i.e., $C_{7+}$, with a measured molecular weight of 150 and specific gravity of 0.78.

Solution

Step 1. Use Equation 1-2 to estimate $T_c$, $p_c$, $V_c$, and $T_b$:

- $T_c = 544.2 \times (150)^{0.2998 \times (0.78)^1.0555 \exp[-1.3478 \times 10^{-4} (150) - 0.61641 (0.78) + 0]} = 1139.4 \ {^\circ}R$
- $p_c = 4.5203 \times 10^4 (150)^{-0.8063 \times (0.78)^{1.6015 \exp[-1.8078 \times 10^{-3} (150) - 0.3084 (0.78) + 0]} = 320.3 \ \text{psia}$
- $V_c = 1.206 \times 10^{-2} (150)^{0.20378 \times (0.78)^{-1.3036 \exp[-2.657 \times 10^{-3} (150) + 0.5287 (0.78) = 2.6012 \times 10^{-3} (150) (0.78)]} = 0.06035 \ \text{ft}^3/\text{lb}$
- $T_b = 6.77857 (150)^{0.401673 \times (0.78)^{-1.58262 \exp[3.77409 \times 10^{-3} (150) + 2.984036 (0.78) - 4.25288 \times 10^{-3} (150) (0.78)]} = 825.26 \ {^\circ}R$

Step 2. Use Edmister’s Equation (Equation 1-3) to estimate the acentric factor:

$$\omega = \frac{3 \log(320.3/14.7)}{7[1139.4/825.26-1]} - 1 = 0.5067$$

PROBLEMS

1. The following is a list of the compositional analysis of different hydrocarbon systems. The compositions are expressed in the terms of mol%.

<table>
<thead>
<tr>
<th>Component</th>
<th>System #1</th>
<th>System #2</th>
<th>System #3</th>
<th>System #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>68.00</td>
<td>25.07</td>
<td>60.00</td>
<td>12.15</td>
</tr>
<tr>
<td>$C_2$</td>
<td>9.68</td>
<td>11.67</td>
<td>8.15</td>
<td>3.10</td>
</tr>
<tr>
<td>$C_3$</td>
<td>5.34</td>
<td>9.36</td>
<td>4.85</td>
<td>2.51</td>
</tr>
<tr>
<td>$C_4$</td>
<td>3.48</td>
<td>6.00</td>
<td>3.12</td>
<td>2.61</td>
</tr>
<tr>
<td>$C_5$</td>
<td>1.78</td>
<td>3.98</td>
<td>1.41</td>
<td>2.78</td>
</tr>
<tr>
<td>$C_6$</td>
<td>1.73</td>
<td>3.26</td>
<td>2.47</td>
<td>4.85</td>
</tr>
<tr>
<td>$C_{7+}$</td>
<td>9.99</td>
<td>40.66</td>
<td>20.00</td>
<td>72.00</td>
</tr>
</tbody>
</table>

Classify these hydrocarbon systems.
2. If a petroleum fraction has a measured molecular weight of 190 and a specific gravity of 0.8762, characterize this fraction by calculating the boiling point, critical temperature, critical pressure, and critical volume of the fraction. Use the Riazi and Daubert correlation.
3. Calculate the acentric factor and critical compressibility factor of the component in the above problem.

REFERENCES

To understand and predict the volumetric behavior of oil and gas reservoirs as a function of pressure, knowledge of the physical properties of reservoir fluids must be gained. These fluid properties are usually determined by laboratory experiments performed on samples of actual reservoir fluids. In the absence of experimentally measured properties, it is necessary for the petroleum engineer to determine the properties from empirically derived correlations. The objective of this chapter is to present several of the well-established physical property correlations for the following reservoir fluids:

- Natural gases
- Crude oil systems
- Reservoir water systems

**PROPERTIES OF NATURAL GASES**

A gas is defined as a homogeneous fluid of low viscosity and density that has no definite volume but expands to completely fill the vessel in which it is placed. Generally, the natural gas is a mixture of hydrocarbon and nonhydrocarbon gases. The hydrocarbon gases that are normally found in a natural gas are methanes, ethanes, propanes, butanes, pentanes, and small amounts of hexanes and heavier. The nonhydrocarbon gases (i.e., impurities) include carbon dioxide, hydrogen sulfide, and nitrogen.
Knowledge of pressure-volume-temperature (PVT) relationships and other physical and chemical properties of gases is essential for solving problems in natural gas reservoir engineering. These properties include:

- Apparent molecular weight, \( M_a \)
- Specific gravity, \( \gamma_g \)
- Compressibility factor, \( z \)
- Density, \( \rho_g \)
- Specific volume, \( v \)
- Isothermal gas compressibility coefficient, \( c_g \)
- Gas formation volume factor, \( B_g \)
- Gas expansion factor, \( E_g \)
- Viscosity, \( \mu_g \)

The above gas properties may be obtained from direct laboratory measurements or by prediction from generalized mathematical expressions. This section reviews laws that describe the volumetric behavior of gases in terms of pressure and temperature and also documents the mathematical correlations that are widely used in determining the physical properties of natural gases.

**BEHAVIOR OF IDEAL GASES**

The kinetic theory of gases postulates that gases are composed of a very large number of particles called molecules. For an ideal gas, the volume of these molecules is insignificant compared with the total volume occupied by the gas. It is also assumed that these molecules have no attractive or repulsive forces between them, and that all collisions of molecules are perfectly elastic.

Based on the above kinetic theory of gases, a mathematical equation called equation-of-state can be derived to express the relationship existing between pressure \( p \), volume \( V \), and temperature \( T \) for a given quantity of moles of gas \( n \). This relationship for perfect gases is called the ideal gas law and is expressed mathematically by the following equation:

\[
pV = nRT \quad (2 - 1)
\]

where

- \( p \) = absolute pressure, psia
- \( V \) = volume, \( \text{ft}^3 \)
- \( T \) = absolute temperature, \( ^\circ\text{R} \)
The number of pound-moles of gas, i.e., $n$, is defined as the weight of the gas $m$ divided by the molecular weight $M$, or:

$$n = \frac{m}{M} \quad (2-2)$$

Combining Equation 2-1 with 2-2 gives:

$$pV = \left(\frac{m}{M}\right)RT \quad (2-3)$$

where $m =$ weight of gas, lb

$\quad M =$ molecular weight, lb/lb-mol

Since the density is defined as the mass per unit volume of the substance, Equation 2-3 can be rearranged to estimate the gas density at any pressure and temperature:

$$\rho_g = \frac{m}{V} = \frac{pM}{RT} \quad (2-4)$$

where $\rho_g =$ density of the gas, lb/ft$^3$

It should be pointed out that lb refers to lbs mass in any of the subsequent discussions of density in this text.

**Example 2-1**

Three pounds of n-butane are placed in a vessel at 120°F and 60 psia. Calculate the volume of the gas assuming an ideal gas behavior.

**Solution**

*Step 1.* Determine the molecular weight of n-butane from Table 1-1 to give:

$$M = 58.123$$
Step 2. Solve Equation 2-3 for the volume of gas:

\[ V = \frac{m}{M} \frac{RT}{p} \]

\[ V = \left( \frac{3}{58.123} \right) \frac{(10.73)(120 + 460)}{60} = 5.35 \text{ ft}^3 \]

Example 2-2

Using the data given in the above example, calculate the density n-butane.

Solution

Solve for the density by applying Equation 2-4:

\[ \rho_g = \frac{(60)(58.123)}{(10.73)(580)} = 0.56 \text{ lb/ft}^3 \]

Petroleum engineers are usually interested in the behavior of mixtures and rarely deal with pure component gases. Because natural gas is a mixture of hydrocarbon components, the overall physical and chemical properties can be determined from the physical properties of the individual components in the mixture by using appropriate mixing rules.

The basic properties of gases are commonly expressed in terms of the apparent molecular weight, standard volume, density, specific volume, and specific gravity. These properties are defined as follows:

**Apparent Molecular Weight**

One of the main gas properties that is frequently of interest to engineers is the apparent molecular weight. If \( y_i \) represents the mole fraction of the \( i \)th component in a gas mixture, the apparent molecular weight is defined mathematically by the following equation:

\[ M_a = \sum_{i=1}^{n} y_i M_i \quad (2 \cdot 5) \]

where

- \( M_a \) = apparent molecular weight of a gas mixture
- \( M_i \) = molecular weight of the \( i \)th component in the mixture
- \( y_i \) = mole fraction of component \( i \) in the mixture
Standard Volume

In many natural gas engineering calculations, it is convenient to measure the volume occupied by 1 lb-mole of gas at a reference pressure and temperature. These reference conditions are usually 14.7 psia and 60°F, and are commonly referred to as standard conditions. The standard volume is then defined as the volume of gas occupied by 1 lb-mol of gas at standard conditions. Applying the above conditions to Equation 2-1 and solving for the volume, i.e., the standard volume, gives:

\[ V_{sc} = \frac{(1) RT_{sc}}{p_{sc}} = \frac{(1)(10.73)(520)}{14.7} \]

or

\[ V_{sc} = 379.4 \text{ scf/lb-mol} \]  \hspace{1cm} (2 - 6)

where \( V_{sc} \) = standard volume, scf/lb-mol
\( scf \) = standard cubic feet
\( T_{sc} \) = standard temperature, °R
\( p_{sc} \) = standard pressure, psia

Density

The density of an ideal gas mixture is calculated by simply replacing the molecular weight of the pure component in Equation 2-4 with the apparent molecular weight of the gas mixture to give:

\[ \rho_g = \frac{pM_a}{RT} \]  \hspace{1cm} (2 - 7)

where \( \rho_g \) = density of the gas mixture, lb/ft³
\( M_a \) = apparent molecular weight

Specific Volume

The specific volume is defined as the volume occupied by a unit mass of the gas. For an ideal gas, this property can be calculated by applying Equation 2-3:

\[ v = \frac{V}{m} = \frac{RT}{pM_a} = \frac{1}{\rho_g} \]  \hspace{1cm} (2 - 8)
where \( v \) = specific volume, \( \text{ft}^3/\text{lb} \)
\( \rho_g \) = gas density, \( \text{lb}/\text{ft}^3 \)

**Specific Gravity**

The specific gravity is defined as the ratio of the gas density to that of the air. Both densities are measured or expressed at the same pressure and temperature. Commonly, the standard pressure \( p_{sc} \) and standard temperature \( T_{sc} \) are used in defining the gas specific gravity:

\[
\gamma_g = \frac{\rho_g}{\rho_{air}}
\]  

(2 - 9)

Assuming that the behavior of both the gas mixture and the air is described by the ideal gas equation, the specific gravity can then be expressed as:

\[
\gamma_g = \frac{p_{sc} M_a}{RT_{sc}} \frac{RT_{sc}}{p_{sc} M_{air}}
\]

or

\[
\gamma_g = \frac{M_a}{M_{air}} = \frac{M_a}{28.96}
\]  

(2 - 10)

where \( \gamma_g \) = gas specific gravity
\( \rho_{air} \) = density of the air
\( M_{air} \) = apparent molecular weight of the air = 28.96
\( M_a \) = apparent molecular weight of the gas
\( p_{sc} \) = standard pressure, psia
\( T_{sc} \) = standard temperature, °R

**Example 2-3**

A gas well is producing gas with a specific gravity of 0.65 at a rate of 1.1 MMscf/day. The average reservoir pressure and temperature are 1,500 psi and 150°F. Calculate:

a. Apparent molecular weight of the gas
b. Gas density at reservoir conditions
c. Flow rate in lb/day
Solution

a. From Equation 2-10, solve for the apparent molecular weight:

\[ M_a = 28.96 \gamma_g \]
\[ M_a = (28.96) (0.65) = 18.82 \]

b. Apply Equation 2-7 to determine gas density:

\[ \rho_g = \frac{(1500)(18.82)}{(10.73)(610)} = 4.31 \text{ lb/ft}^3 \]

c. Step 1. Because 1 lb-mol of any gas occupies 379.4 scf at standard conditions, then the daily number of moles that the gas well is producing can be calculated from:

\[ n = \frac{(1.1)(10)^6}{379.4} = 2899 \text{ lb-mol} \]

Step 2. Determine the daily mass \( m \) of the gas produced from Equation 2-2:

\[ m = (n)(M_a) \]
\[ m = (2899)(18.82) = 54559 \text{ lb/day} \]

Example 2-4

A gas well is producing a natural gas with the following composition:

<table>
<thead>
<tr>
<th>Component</th>
<th>( \gamma_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO(_2)</td>
<td>0.05</td>
</tr>
<tr>
<td>C(_1)</td>
<td>0.90</td>
</tr>
<tr>
<td>C(_2)</td>
<td>0.03</td>
</tr>
<tr>
<td>C(_3)</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Assuming an ideal gas behavior, calculate:

a. Apparent molecular weight  
b. Specific gravity  
c. Gas density at 2000 psia and 150°F  
d. Specific volume at 2000 psia and 150°F  

**Solution**

<table>
<thead>
<tr>
<th>Component</th>
<th>( \gamma_i )</th>
<th>( M_i )</th>
<th>( \gamma_i \cdot M_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO(_2)</td>
<td>0.05</td>
<td>44.01</td>
<td>2.200</td>
</tr>
<tr>
<td>C(_1)</td>
<td>0.90</td>
<td>16.04</td>
<td>14.436</td>
</tr>
<tr>
<td>C(_2)</td>
<td>0.03</td>
<td>30.07</td>
<td>0.902</td>
</tr>
<tr>
<td>C(_3)</td>
<td>0.02</td>
<td>44.11</td>
<td>0.882</td>
</tr>
</tbody>
</table>

\[ M_a = 18.42 \]

a. Apply Equation 2-5 to calculate the apparent molecular weight:

\[ M_a = 18.42 \]

b. Calculate the specific gravity by using Equation 2-10:

\[ \gamma_g = \frac{18.42}{28.96} = 0.636 \]

c. Solve for the density by applying Equation 2-7:

\[ \rho_g = \frac{(2000)(18.42)}{(10.73)(610)} = 5.628 \text{ lb/ft}^3 \]

d. Determine the specific volume from Equation 2-8:

\[ v = \frac{1}{5.628} = 0.178 \text{ ft}^3/\text{lb} \]

**BEHAVIOR OF REAL GASES**

In dealing with gases at a very low pressure, the ideal gas relationship is a convenient and generally satisfactory tool. At higher pressures, the use of the ideal gas equation-of-state may lead to errors as great as 500%, as compared to errors of 2–3% at atmospheric pressure.
Basically, the magnitude of deviations of real gases from the conditions of the ideal gas law increases with increasing pressure and temperature and varies widely with the composition of the gas. Real gases behave differently than ideal gases. The reason for this is that the perfect gas law was derived under the assumption that the volume of molecules is insignificant and that no molecular attraction or repulsion exists between them. This is not the case for real gases.

Numerous equations-of-state have been developed in the attempt to correlate the pressure-volume-temperature variables for real gases with experimental data. In order to express a more exact relationship between the variables $p$, $V$, and $T$, a correction factor called the gas compressibility factor, gas deviation factor, or simply the $z$-factor, must be introduced into Equation 2-1 to account for the departure of gases from ideality. The equation has the following form:

$$pV = znRT$$

(2-11)

where the gas compressibility factor $z$ is a dimensionless quantity and is defined as the ratio of the actual volume of $n$-moles of gas at $T$ and $p$ to the ideal volume of the same number of moles at the same $T$ and $p$:

$$z = \frac{V_{\text{actual}}}{V_{\text{ideal}}} = \frac{V}{(nRT)/p}$$

Studies of the gas compressibility factors for natural gases of various compositions have shown that compressibility factors can be generalized with sufficient accuracies for most engineering purposes when they are expressed in terms of the following two dimensionless properties:

- Pseudo-reduced pressure
- Pseudo-reduced temperature

These dimensionless terms are defined by the following expressions:

$$p_{pr} = \frac{p}{p_{pc}}$$

(2 - 12)

$$T_{pr} = \frac{T}{T_{pc}}$$

(2 - 13)
where \( p \) = system pressure, psia

\( p_{pr} \) = pseudo-reduced pressure, dimensionless

\( T \) = system temperature, °R

\( T_{pr} \) = pseudo-reduced temperature, dimensionless

\( p_{pc}, T_{pc} \) = pseudo-critical pressure and temperature, respectively, and defined by the following relationships:

\[
p_{pc} = \sum_{i=1}^{n} y_i p_{ci} \quad (2-14)
\]

\[
T_{pc} = \sum_{i=1}^{n} y_i T_{ci} \quad (2-15)
\]

It should be pointed out that these pseudo-critical properties, i.e., \( p_{pc} \) and \( T_{pc} \), do not represent the actual critical properties of the gas mixture. These pseudo properties are used as correlating parameters in generating gas properties.

Based on the concept of pseudo-reduced properties, Standing and Katz (1942) presented a generalized gas compressibility factor chart as shown in Figure 2-1. The chart represents compressibility factors of sweet natural gas as a function of \( p_{pr} \) and \( T_{pr} \). This chart is generally reliable for natural gas with minor amount of nonhydrocarbons. It is one of the most widely accepted correlations in the oil and gas industry.

**Example 2-5**

A gas reservoir has the following gas composition: the initial reservoir pressure and temperature are 3000 psia and 180°F, respectively.

<table>
<thead>
<tr>
<th>Component</th>
<th>( y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>0.02</td>
</tr>
<tr>
<td>N₂</td>
<td>0.01</td>
</tr>
<tr>
<td>C₁</td>
<td>0.85</td>
</tr>
<tr>
<td>C₂</td>
<td>0.04</td>
</tr>
<tr>
<td>C₃</td>
<td>0.03</td>
</tr>
<tr>
<td>i - C₄</td>
<td>0.03</td>
</tr>
<tr>
<td>n - C₄</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Calculate the gas compressibility factor under initial reservoir conditions.
Figure 2-1. Standing and Katz compressibility factors chart. (Courtesy of GPSA and GPA Engineering Data Book, EO Edition, 1987.)
Solution

<table>
<thead>
<tr>
<th>Component</th>
<th>$\gamma_i$</th>
<th>$T_{ci}^°R$</th>
<th>$\gamma_iT_{ci}$</th>
<th>$p_{ci}$</th>
<th>$\gamma_iP_{ci}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>0.02</td>
<td>547.91</td>
<td>10.96</td>
<td>1071</td>
<td>21.42</td>
</tr>
<tr>
<td>N₂</td>
<td>0.01</td>
<td>227.49</td>
<td>2.27</td>
<td>493.1</td>
<td>4.93</td>
</tr>
<tr>
<td>C₁</td>
<td>0.85</td>
<td>343.33</td>
<td>291.83</td>
<td>666.4</td>
<td>566.44</td>
</tr>
<tr>
<td>C₂</td>
<td>0.04</td>
<td>549.92</td>
<td>22.00</td>
<td>706.5</td>
<td>28.26</td>
</tr>
<tr>
<td>C₃</td>
<td>0.03</td>
<td>666.06</td>
<td>19.98</td>
<td>616.4</td>
<td>18.48</td>
</tr>
<tr>
<td>i - C₄</td>
<td>0.03</td>
<td>734.46</td>
<td>22.03</td>
<td>527.9</td>
<td>15.84</td>
</tr>
<tr>
<td>n - C₄</td>
<td>0.02</td>
<td>765.62</td>
<td>15.31</td>
<td>550.6</td>
<td>11.01</td>
</tr>
</tbody>
</table>

$T_{pc} = 383.38 \quad p_{pc} = 666.38$

Step 1. Determine the pseudo-critical pressure from Equation 2-14:

$p_{pc} = 666.18$

Step 2. Calculate the pseudo-critical temperature from Equation 2-15:

$T_{pc} = 383.38$

Step 3. Calculate the pseudo-reduced pressure and temperature by applying Equations 2-12 and 2-13, respectively:

$p_{pr} = \frac{3000}{666.38} = 4.50$

$T_{pr} = \frac{640}{383.38} = 1.67$

Step 4. Determine the z-factor from Figure 2-1, to give:

$z = 0.85$

Equation 2-11 can be written in terms of the apparent molecular weight $M_a$ and the weight of the gas $m$:

$pV = z \left( \frac{m}{M_a} \right) RT$

Solving the above relationship for the gas specific volume and density, give:
\[ v = \frac{V}{m} = \frac{zRT}{pM_a} \]  
\[ \rho_g = \frac{1}{v} = \frac{pM_a}{zRT} \]

where \( v \) = specific volume, \( \text{ft}^3/\text{lb} \)
\( \rho_g \) = density, \( \text{lb/ft}^3 \)

**Example 2-6**

Using the data in Example 2-5 and assuming real gas behavior, calculate the density of the gas phase under initial reservoir conditions. Compare the results with that of ideal gas behavior.

**Solution**

<table>
<thead>
<tr>
<th>Component</th>
<th>( y_i )</th>
<th>( M_i )</th>
<th>( y_i \cdot M_i )</th>
<th>( T_{ci},^\circ\text{R} )</th>
<th>( \frac{y_i T_{ci}}{p_\text{ci}} )</th>
<th>( p_\text{ci} )</th>
<th>( y_i p_\text{ci} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO(_2)</td>
<td>0.02</td>
<td>44.01</td>
<td>0.88</td>
<td>547.91</td>
<td>10.96</td>
<td>1071</td>
<td>21.42</td>
</tr>
<tr>
<td>N(_2)</td>
<td>0.01</td>
<td>28.01</td>
<td>0.28</td>
<td>227.49</td>
<td>2.27</td>
<td>493.1</td>
<td>4.93</td>
</tr>
<tr>
<td>C(_1)</td>
<td>0.85</td>
<td>16.04</td>
<td>13.63</td>
<td>343.33</td>
<td>291.83</td>
<td>666.4</td>
<td>566.44</td>
</tr>
<tr>
<td>C(_2)</td>
<td>0.04</td>
<td>30.1</td>
<td>1.20</td>
<td>549.92</td>
<td>22.00</td>
<td>706.5</td>
<td>28.26</td>
</tr>
<tr>
<td>C(_3)</td>
<td>0.03</td>
<td>44.1</td>
<td>1.32</td>
<td>666.06</td>
<td>19.98</td>
<td>616.40</td>
<td>18.48</td>
</tr>
<tr>
<td>1 - C(_4)</td>
<td>0.03</td>
<td>58.1</td>
<td>1.74</td>
<td>734.46</td>
<td>22.03</td>
<td>527.9</td>
<td>15.84</td>
</tr>
<tr>
<td>n - C(_4)</td>
<td>0.02</td>
<td>58.1</td>
<td>1.16</td>
<td>765.62</td>
<td>15.31</td>
<td>550.6</td>
<td>11.01</td>
</tr>
</tbody>
</table>

\[ M_a = 20.23 \]
\[ T_{pc} = 383.38 \]
\[ P_{pc} = 666.38 \]

**Step 1.** Calculate the apparent molecular weight from Equation 2-5:
\[ M_a = 20.23 \]

**Step 2.** Determine the pseudo-critical pressure from Equation 2-14:
\[ p_{pc} = 666.18 \]

**Step 3.** Calculate the pseudo-critical temperature from Equation 2-15:
\[ T_{pc} = 383.38 \]

**Step 4.** Calculate the pseudo-reduced pressure and temperature by applying Equations 2-12 and 2-13, respectively:
Step 5. Determine the z-factor from Figure 2-1:

\[ z = 0.85 \]

Step 6. Calculate the density from Equation 2-17:

\[ \rho_g = \frac{(3000)(20.23)}{(0.85)(10.73)(640)} = 10.4 \text{ lb/ft}^3 \]

Step 7. Calculate the density of the gas assuming an ideal gas behavior from Equation 2-7:

\[ \rho_g = \frac{(3000)(20.23)}{(10.73)(640)} = 8.84 \text{ lb/ft}^3 \]

The results of the above example show that the ideal gas equation estimated the gas density with an absolute error of 15% when compared with the density value as predicted with the real gas equation.

In cases where the composition of a natural gas is not available, the pseudo-critical properties, i.e., \( p_{pc} \) and \( T_{pc} \), can be predicted solely from the specific gravity of the gas. Brown et al. (1948) presented a graphical method for a convenient approximation of the pseudo-critical pressure and pseudo-critical temperature of gases when only the specific gravity of the gas is available. The correlation is presented in Figure 2-2. Standing (1977) expressed this graphical correlation in the following mathematical forms:

**Case 1: Natural Gas Systems**

\[
\begin{align*}
T_{pc} &= 168 + 325 \gamma_k - 12.5 \gamma_k^2 \\
p_{pc} &= 677 + 15.0 \gamma_k - 37.5 \gamma_k^2
\end{align*}
\]

**Case 2: Gas-Condensate Systems**

\[
\begin{align*}
T_{pc} &= 187 + 330 \gamma_k - 71.5 \gamma_k^2
\end{align*}
\]
where \( T_{pc} \) = pseudo-critical temperature, °R
\( p_{pc} \) = pseudo-critical pressure, psia
\( \gamma_g \) = specific gravity of the gas mixture

**Example 2-7**

Rework Example 2-5 by calculating the pseudo-critical properties from Equations 2-18 and 2-19.
Solution

*Step 1.* Calculate the specific gravity of the gas:

\[ \gamma_g = \frac{M_g}{28.96} = \frac{20.23}{28.96} = 0.699 \]

*Step 2.* Solve for the pseudo-critical properties by applying Equations 2-18 and 2-19:

\[ T_{pc} = 168 + 325 (0.699) - 12.5 (0.699)^2 = 389.1^\circ R \]

\[ p_{pc} = 677 + 15 (0.699) - 37.5 (0.699)^2 = 669.2 \text{ psia} \]

*Step 3.* Calculate \( p_{pr} \) and \( T_{pr} \):

\[ p_{pr} = \frac{3000}{669.2} = 4.48 \]

\[ T_{pr} = \frac{640}{389.1} = 1.64 \]

*Step 4.* Determine the gas compressibility factor from Figure 2-1:

\[ z = 0.824 \]

*Step 5.* Calculate the density from Equation 2-17:

\[ \rho_g = \frac{(3000)(20.23)}{(0.845)(10.73)(640)} = 10.46 \text{ lb/ft}^3 \]

**EFFECT OF NONHYDROCARBON COMPONENTS ON THE Z-FACTOR**

Natural gases frequently contain materials other than hydrocarbon components, such as nitrogen, carbon dioxide, and hydrogen sulfide. Hydrocarbon gases are classified as sweet or sour depending on the hydrogen sulfide content. Both sweet and sour gases may contain nitrogen, carbon dioxide, or both. A hydrocarbon gas is termed a sour gas if it contains one grain of H\(_2\)S per 100 cubic feet.

The common occurrence of small percentages of nitrogen and carbon dioxide is, in part, considered in the correlations previously cited. Con-
centrations of up to 5 percent of these nonhydrocarbon components will not seriously affect accuracy. Errors in compressibility factor calculations as large as 10 percent may occur in higher concentrations of nonhydrocarbon components in gas mixtures.

**Nonhydrocarbon Adjustment Methods**

There are two methods that were developed to adjust the pseudo-critical properties of the gases to account for the presence of the nonhydrocarbon components. These two methods are the:

- Wichert-Aziz correction method
- Carr-Kobayashi-Burrows correction method

**The Wichert-Aziz Correction Method**

Natural gases that contain H₂S and or CO₂ frequently exhibit different compressibility-factors behavior than do sweet gases. Wichert and Aziz (1972) developed a simple, easy-to-use calculation procedure to account for these differences. This method permits the use of the Standing-Katz chart, i.e., Figure 2-1, by using a pseudo-critical temperature adjustment factor, which is a function of the concentration of CO₂ and H₂S in the sour gas. This correction factor is then used to adjust the pseudo-critical temperature and pressure according to the following expressions:

\[
T'_{pc} = T_{pc} - \varepsilon
\]

\[
p'_{pc} = \frac{p_{pc} T'_{pc}}{T_{pc} + B (1 - B) \varepsilon}
\]

where
- \( T_{pc} \) = pseudo-critical temperature, °R
- \( p_{pc} \) = pseudo-critical pressure, psia
- \( T'_{pc} \) = corrected pseudo-critical temperature, °R
- \( p'_{pc} \) = corrected pseudo-critical pressure, psia
- \( B \) = mole fraction of H₂S in the gas mixture
- \( \varepsilon \) = pseudo-critical temperature adjustment factor and is defined mathematically by the following expression

\[
\varepsilon = 120 \ [A^{0.9} - A^{1.6}] + 15 \ (B^{0.5} - B^{4.0})
\]

where the coefficient A is the sum of the mole fraction H₂S and CO₂ in the gas mixture, or:
\[ A = y_{H_2S} + y_{CO_2} \]

The computational steps of incorporating the adjustment factor \( e \) into the \( z \)-factor calculations are summarized below:

**Step 1.** Calculate the pseudo-critical properties of the whole gas mixture by applying Equations 2-18 and 2-19 or Equations 2-20 and 2-21.

**Step 2.** Calculate the adjustment factor \( e \) from Equation 2-24.

**Step 3.** Adjust the calculated \( p_{pc} \) and \( T_{pc} \) (as computed in Step 1) by applying Equations 2-22 and 2-23.

**Step 4.** Calculate the pseudo-reduced properties, i.e., \( p_{pr} \) and \( T_{pr} \), from Equations 2-11 and 2-12.

**Step 5.** Read the compressibility factor from Figure 2-1.

**Example 2-8**

A sour natural gas has a specific gravity of 0.7. The compositional analysis of the gas shows that it contains 5 percent CO\(_2\) and 10 percent H\(_2\)S. Calculate the density of the gas at 3500 psia and 160\(^\circ\)F.

**Solution**

**Step 1.** Calculate the uncorrected pseudo-critical properties of the gas from Equations 2-18 and 2-19:

\[ T_{pc} = 168 + 325 (0.7) - 12.5 (0.7)^2 = 389.38^\circ R \]
\[ p_{pc} = 677 + 15 (0.7) - 37.5 (0.7)^2 = 669.1 \text{ psia} \]

**Step 2.** Calculate the pseudo-critical temperature adjustment factor from Equation 2-24:

\[ e = 120 (0.15^{0.9} - 0.15^{1.6}) + 15 (0.1^{0.5} - 0.1^4) = 20.735 \]

**Step 3.** Calculate the corrected pseudo-critical temperature by applying Equation 2-22:
Step 4. Adjust the pseudo-critical pressure $p_{pc}$ by applying Equation 2.23:

$$p'_{pc} = \frac{(669.1)(368.64)}{389.38 + 0.1(1-0.1)(20.635)}$$

Step 5. Calculate $p_{pr}$ and $T_{pr}$:

$$p_{pr} = \frac{3500}{630.44} = 5.55$$

$$T_{pr} = \frac{160 + 460}{368.64} = 1.68$$

Step 6. Determine the $z$-factor from Figure 2-1:

$$z = 0.89$$

Step 7. Calculate the apparent molecular weight of the gas from Equation 2.10:

$$M_a = (28.96)(0.7) = 20.27$$

Step 8. Solve for gas density:

$$\rho_g = \frac{(3500)(20.27)}{(0.89)(10.73)(620)} = 11.98 \text{ lb/ft}^3$$

**The Carr-Kobayashi-Burrows Correction Method**

Carr, Kobayashi, and Burrows (1954) proposed a simplified procedure to adjust the pseudo-critical properties of natural gases when nonhydrocarbon components are present. The method can be used when the composition of the natural gas is not available. The proposed procedure is summarized in the following steps:

Step 1. Knowing the specific gravity of the natural gas, calculate the pseudo-critical temperature and pressure by applying Equations 2-18 and 2-19.
Step 2. Adjust the estimated pseudo-critical properties by using the following two expressions:

\[
T'_{pc} = T_{pc} - 80 y_{CO_2} + 130 y_{H_2S} - 250 y_{N_2} \tag{2-25}
\]

\[
p'_{pc} = p_{pc} + 440 y_{CO_2} + 600 y_{H_2S} - 170 y_{N_2} \tag{2-26}
\]

where \( T'_{pc} \) = the adjusted pseudo-critical temperature, °R
\( T_{pc} \) = the unadjusted pseudo-critical temperature, °R
\( y_{CO_2} \) = mole fraction of CO\(_2\)
\( y_{N_2} \) = mole fraction of H\(_2\)S in the gas mixture
\( y_{N_2} \) = mole fraction of Nitrogen
\( p'_{pc} \) = the adjusted pseudo-critical pressure, psia
\( p_{pc} \) = the unadjusted pseudo-critical pressure, psia

Step 3. Use the adjusted pseudo-critical temperature and pressure to calculate the pseudo-reduced properties.

Step 4. Calculate the \( z \)-factor from Figure 2-1.

Example 2-9

Using the data in Example 2-8, calculate the density by employing the above correction procedure.

Solution

Step 1. Determine the corrected pseudo-critical properties from Equations 2-25 and 2-26:

\[
T'_{pc} = 389.38 - 80 (0.05) + 130 (0.10) - 250 (0) = 398.38°R
\]

\[
p'_{pc} = 669.1 + 440 (0.05) + 600 (0.10) - 170 (0) = 751.1 \text{ psia}
\]

Step 2. Calculate \( \rho_{pr} \) and \( T_{pr} \):

\[
\rho_{pr} = \frac{3500}{751.1} = 4.56
\]

\[
T_{pr} = \frac{620}{398.38} = 1.56
\]
Step 3. Determine the gas compressibility factor from Figure 2-1:

\[ z = 0.820 \]

Step 4. Calculate the gas density:

\[ \rho_g = \frac{(3500)(20.27)}{(0.82)(10.73)(620)} = 13.0 \text{ lb/ft}^3 \]

**CORRECTION FOR HIGH-MOLECULAR WEIGHT GASES**

It should be noted that the Standing and Katz compressibility factor chart (Figure 2-1) was prepared from data on binary mixtures of methane with propane, ethane, and butane, and on natural gases, thus covering a wide range in composition of hydrocarbon mixtures containing methane. No mixtures having molecular weights in excess of 40 were included in preparing this plot.

Sutton (1985) evaluated the accuracy of the Standing-Katz compressibility factor chart using laboratory-measured gas compositions and z-factors, and found that the chart provides satisfactory accuracy for engineering calculations. However, Kay’s mixing rules, i.e., Equations 2-13 and 2-14 (or comparable gravity relationships for calculating pseudo-critical pressure and temperature), result in unsatisfactory z-factors for high molecular weight reservoir gases. The author observed that large deviations occur to gases with high heptanes-plus concentrations. He pointed out that Kay’s mixing rules should not be used to determine the pseudo-critical pressure and temperature for reservoir gases with specific gravities greater than about 0.75.

Sutton proposed that this deviation can be minimized by utilizing the mixing rules developed by Stewart et al. (1959), together with newly introduced empirical adjustment factors (FJ, EJ, and EK) that are related to the presence of the heptane-plus fraction in the gas mixture. The proposed approach is outlined in the following steps:

**Step 1.** Calculate the parameters J and K from the following relationships:

\[ J = \frac{1}{3} \left[ \sum \frac{y_i (T_{ci}/p_{ci})}{yi (T_{ci}/p_{ci})} \right] + \frac{2}{3} \left[ \sum \frac{y_i (T_{ci}/p_{ci})^{0.5}}{yi (T_{ci}/p_{ci})^{0.5}} \right]^2 \]  

\[ (2-27) \]
where $J = \text{Stewart-Burkhardt-Voo correlating parameter}, \degree R/\text{psia}$

$K = \text{Stewart-Burkhardt-Voo correlating parameter}, \degree R/\text{psia}$

$y_i = \text{mole fraction of component } i \text{ in the gas mixture.}$

**Step 2.** Calculate the adjustment parameters $F_J$, $E_J$, and $E_K$ from the following expressions:

$$E_J = 0.6081 F_J + 1.1325 F_J^2 - 14.004 F_J y_{C7^+} + 64.434 F_J y^2_{C7^+}$$

$$E_K = \left[\frac{T_c}{(p_c)_{C7^+}}\right] y_{C7^+} + \left[0.3129 y_{C7^+} - 4.8156 (y_{C7^+})^2 + 27.3751 (y_{C7^+})^3\right]$$

where $y_{C7^+} = \text{mole fraction of the heptanes-plus component}$

$(T_c)_{C7^+} = \text{critical temperature of the } C_{7^+}$

$(p_c)_{C7^+} = \text{critical pressure of the } C_{7^+}$

**Step 3.** Adjust the parameters $J$ and $K$ by applying the adjustment factors $E_J$ and $E_K$, according to the relationships:

$$J' = J - E_J$$

$$K' = K - E_K$$

where $J, K = \text{calculated from Equations 2-27 and 2-28}$

$E_J, E_K = \text{calculated from Equations 2-30 and 2-31}$

**Step 4.** Calculate the adjusted pseudo-critical temperature and pressure from the expressions:

$$T_{pc}' = \frac{(K')^2}{J'}$$

$$p_{pc}' = \frac{T_{pc}'}{J'}$$
Step 5. Having calculated the adjusted $T_{pc}$ and $p_{pc}$, the regular procedure of calculating the compressibility factor from the Standing and Katz chart is followed.

Sutton’s proposed mixing rules for calculating the pseudo-critical properties of high-molecular-weight reservoir gases, i.e., $\gamma_g > 0.75$, should significantly improve the accuracy of the calculated $z$-factor.

Example 2-10

A hydrocarbon gas system has the following composition:

<table>
<thead>
<tr>
<th>Component</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.83</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.06</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.03</td>
</tr>
<tr>
<td>$n-C_4$</td>
<td>0.02</td>
</tr>
<tr>
<td>$n-C_5$</td>
<td>0.02</td>
</tr>
<tr>
<td>$C_6$</td>
<td>0.01</td>
</tr>
<tr>
<td>$C_{7+}$</td>
<td>0.03</td>
</tr>
</tbody>
</table>

The heptanes-plus fraction is characterized by a molecular weight and specific gravity of 161 and 0.81, respectively.

a. Using Sutton’s methodology, calculate the density of the gas 2000 psi and 150°F.
b. Recalculate the gas density without adjusting the pseudo-critical properties.

Solution

Part A.

Step 1. Calculate the critical properties of the heptanes-plus fraction by the Riazi-Daubert correlation (Chapter 1, Equation 1-2):

$$
(T_c)_{C_{7+}} = 544.2\ 161^{0.29980.81^{1.0555}} \exp\left[-1.3478(10^{-4} (150)-0.61641(0.81))\right] = 1189\,^\circ R
$$

$$
(p_c)_{C_{7+}} = 4.5203(10)^4\ 161^{-0.80630.81^{1.6015}} \exp\left[-1.8078(10^{-5} (150)-0.3084(0.81))\right] = 318.4\,\text{psia}
$$
Step 2. Construct the following table:

<table>
<thead>
<tr>
<th>Component</th>
<th>$y_i$</th>
<th>$M_i$</th>
<th>$T_{ci}$</th>
<th>$p_{ci}$</th>
<th>$y_iM_i$</th>
<th>$y_i[T_{ci}/p_{ci}]$</th>
<th>$y_i[T_{ci}/z_{pc}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.83</td>
<td>16.0</td>
<td>343.33</td>
<td>666.4</td>
<td>13.31</td>
<td>.427</td>
<td>.596</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.06</td>
<td>30.1</td>
<td>549.92</td>
<td>706.5</td>
<td>1.81</td>
<td>.047</td>
<td>.053</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.03</td>
<td>44.1</td>
<td>666.06</td>
<td>616.4</td>
<td>1.32</td>
<td>.032</td>
<td>.031</td>
</tr>
<tr>
<td>n-$C_4$</td>
<td>0.02</td>
<td>58.1</td>
<td>765.62</td>
<td>550.6</td>
<td>1.16</td>
<td>.028</td>
<td>.024</td>
</tr>
<tr>
<td>n-$C_5$</td>
<td>0.02</td>
<td>72.2</td>
<td>845.60</td>
<td>488.6</td>
<td>1.45</td>
<td>.035</td>
<td>.026</td>
</tr>
<tr>
<td>$C_6$</td>
<td>0.01</td>
<td>84.0</td>
<td>923.00</td>
<td>483.0</td>
<td>0.84</td>
<td>.019</td>
<td>.014</td>
</tr>
<tr>
<td>$C_7^*$</td>
<td>0.03</td>
<td>161.</td>
<td>1189.0</td>
<td>318.4</td>
<td>4.83</td>
<td>.112</td>
<td>.058</td>
</tr>
<tr>
<td>Total</td>
<td>27.72</td>
<td>0.700</td>
<td>0.802</td>
<td>16.972</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 3. Calculate the parameters $J$ and $K$ from Equations 2-27 and 2-28:

$$J = \left(\frac{1}{3}\right) [0.700] + \left(\frac{2}{3}\right) [0.802]^2 = 0.662$$

$K = 16.922$

Step 4. Determine the adjustment factors $F_J$, $E_J$ and $E_K$ by applying Equations 2-29 through 2-31:

$$F_J = \frac{1}{3} [0.112] + \frac{2}{3} [0.058]^2 = 0.0396$$

$$E_J = 0.6081 (0.04) + 1.1325 (0.04)^2 - 14.004 (0.04) (0.03) + 64.434 (0.04) 0.3^2 = 0.012$$

$$E_K = 66.634 [0.3129 (0.03) - 4.8156 (0.03)^2 + 27.3751 (0.03)^3] = 0.386$$

Step 5. Calculate the parameters $J'$ and $K'$ from Equations 2-32 and 2-33:

$$J' = 0.662 - 0.012 = 0.650$$

$$K' = 16.922 - 0.386 = 16.536$$

Step 6. Determine the adjusted pseudo-critical properties from Equations 2-33 and 2-36:

$$T_{pc}' = \frac{(16.536)^2}{0.65} = 420.7$$
Step 7. Calculate the pseudo-reduced properties of the gas by applying Equations 2-11 and 2-12, to give:

\[ p_{pr} = \frac{420.7}{0.65} = 647.2 \]

\[ T_{pr} = \frac{610}{420.7} = 1.45 \]

Step 8. Calculate the z-factor from Figure 2-1, to give:

\[ z = 0.745 \]

Step 9. From Equation 2-16, calculate the density of the gas:

\[ \rho_g = \frac{(2000)(24.73)}{(10.73)(610)(0.745)} = 10.14 \text{ lb/ft}^3 \]

Part B.

Step 1. Calculate the specific gravity of the gas:

\[ \gamma_g = \frac{M_n}{28.96} = \frac{24.73}{28.96} = 0.854 \]

Step 2. Solve for the pseudo-critical properties by applying Equations 2-18 and 2-19:

\[ T_{pc} = 168 + 325 (0.854) - 12.5 (0.854)^2 = 436.4^\circ R \]

\[ p_{pc} = 677 + 15 (0.854) - 37.5 (0.854)^2 = 662.5 \text{ psia} \]

Step 3. Calculate \( p_{pr} \) and \( T_{pr} \):

\[ p_{pr} = \frac{2000}{662.5} = 3.02 \]

\[ T_{pr} = \frac{610}{436.4} = 1.40 \]
Step 4. Calculate the $z$-factor from Figure 2-1, to give:

$$z = 0.710$$

Step 5. From Equation 2-16, calculate the density of the gas:

$$\rho_g = \frac{(2000)(24.73)}{(10.73)(610)(.710)} = 10.64 \text{lb/ft}^3$$

**DIRECT CALCULATION OF COMPRESSIBILITY FACTORS**

After four decades of existence, the Standing-Katz $z$-factor chart is still widely used as a practical source of natural gas compressibility factors. As a result, there has been an apparent need for a simple mathematical description of that chart. Several empirical correlations for calculating $z$-factors have been developed over the years. The following three empirical correlations are described below:

- Hall-Yarborough
- Dranchuk-Abu-Kassem
- Dranchuk-Purvis-Robinson

**The Hall-Yarborough Method**

Hall and Yarborough (1973) presented an equation-of-state that accurately represents the Standing and Katz $z$-factor chart. The proposed expression is based on the Starling-Carnahan equation-of-state. The coefficients of the correlation were determined by fitting them to data taken from the Standing and Katz $z$-factor chart. Hall and Yarborough proposed the following mathematical form:

$$z = \left[ \frac{0.06125 \rho_{pr} t}{Y} \right] \exp [-1.2(1-t)^2]$$  \hspace{1cm} (2 - 36)

where
- $\rho_{pr} =$ pseudo-reduced pressure
- $t =$ reciprocal of the pseudo-reduced temperature, i.e., $T_{pc}/T$
- $Y =$ the reduced density that can be obtained as the solution of the following equation:
F(Y) = X1 + \frac{Y + Y^2 + Y^3 + Y^4}{(1 - Y)^3} - (X2) Y^2 + (X3) Y^X4 = 0 \quad (2 - 37)

where X1 = -0.06125 ppr t \exp[-1.2 (1 - t)^2]
X2 = (14.76 t - 9.76 t^2 + 4.58 t^3)
X3 = (90.7 t - 242.2 t^2 + 42.4 t^3)
X4 = (2.18 + 2.82 t)

Equation 2-37 is a nonlinear equation and can be conveniently solved for the reduced density Y by using the Newton-Raphson iteration technique. The computational procedure of solving Equation 2-37 at any specified pseudo-reduced pressure ppr and temperature Tpr is summarized in the following steps:

Step 1. Make an initial guess of the unknown parameter, Yk, where k is an iteration counter. An appropriate initial guess of Y is given by the following relationship:

\[ Y^k = 0.0125 \ \text{ppr} \ t \exp[-1.2 (1 - t)^2] \]

Step 2. Substitute this initial value in Equation 2-37 and evaluate the nonlinear function. Unless the correct value of Y has been initially selected, Equation 2-37 will have a nonzero value of F(Y):

Step 3. A new improved estimate of Y, i.e., Y^{k+1}, is calculated from the following expression:

\[ Y^{k+1} = Y^k - \frac{f(Y^k)}{f'(Y^k)} \quad (2 - 38) \]

where f'(Y^k) is obtained by evaluating the derivative of Equation 2-37 at Y^k, or:

\[ f'(Y) = \frac{1 + 4Y + 4Y^2 - 4Y^3 + Y^4}{(1 - Y)^4} - 2 (X2) Y \]
\[ + (X3) (X4) Y^{(X4-1)} \quad (2 - 39) \]

Step 4. Steps 2–3 are repeated n times, until the error, i.e., abs(Y^k – Y^{k+1}), becomes smaller than a preset tolerance, e.g., 10^{-12}:
**Step 5.** The correct value of $Y$ is then used to evaluate Equation 2-36 for the compressibility factor.

Hall and Yarborough pointed out that the method is not recommended for application if the pseudo-reduced temperature is less than one.

**The Dranchuk-Abu-Kassem Method**

Dranchuk and Abu-Kassem (1975) derived an analytical expression for calculating the reduced gas density that can be used to estimate the gas compressibility factor. The reduced gas density $\rho_r$ is defined as the ratio of the gas density at a specified pressure and temperature to that of the gas at its critical pressure or temperature, or:

$$
\rho_r = \frac{\rho}{\rho_c} = \frac{p M_a / [zT]}{p_c M_a / [z_c R T_c]} = \frac{p / [zT]}{p_c / [z_c T_c]}
$$

The critical gas compressibility factor $z_c$ is approximately 0.27 which leads to the following simplified expression for the reduced gas density:

$$
\rho_r = \frac{0.27}{Z T_{pr}} \rho_{pr} \quad (2 \cdot 40)
$$

The authors proposed the following eleven-constant equation-of-state for calculating the reduced gas density:

$$
f (\rho_r) = \left( R_1 \right) \frac{\rho_r - R_2}{\rho_r} - \left( R_3 \right) \rho_r^2 - \left( R_4 \right) \rho_r^3 + \left( R_5 \right) (1 + A_{11} \rho_r^2) \exp \left[ -A_{11} \rho_r^2 \right] + 1 = 0 \quad (2 \cdot 41)
$$

With the coefficients $R_1$ through $R_5$ as defined by the following relations:

$$
R_1 = \left[ A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_5}{T_{pr}^5} \right]
$$

$$
R_2 = \left[ \frac{0.27}{T_{pr}} \rho_{pr} \right]
$$
The constants $A_1$ through $A_{11}$ were determined by fitting the equation, using nonlinear regression models, to 1,500 data points from the Standing and Katz z-factor chart. The coefficients have the following values:

\[
\begin{align*}
A_1 &= 0.3265 & A_2 &= -1.0700 & A_3 &= -0.5339 & A_4 &= 0.01569 \\
A_5 &= -0.05165 & A_6 &= 0.5475 & A_7 &= -0.7361 & A_8 &= 0.1844 \\
A_9 &= 0.1056 & A_{10} &= 0.6134 & A_{11} &= 0.7210
\end{align*}
\]

Equation 2-41 can be solved for the reduced gas density $\rho_r$ by applying the Newton-Raphson iteration technique as summarized in the following steps:

1. **Step 1.** Make an initial guess of the unknown parameter, $\rho_r^k$, where $k$ is an iteration counter. An appropriate initial guess of $\rho_r^k$ is given by the following relationship:

\[
\rho_r = \frac{0.27 \, p_{pr}}{T_{pr}}
\]

2. **Step 2.** Substitute this initial value in Equation 2-41 and evaluate the nonlinear function. Unless the correct value of $\rho_r^k$ has been initially selected, Equation 2-41 will have a nonzero value for the function $f(\rho_r^k)$.

3. **Step 3.** A new improved estimate of $\rho_r$, i.e., $\rho_r^{k+1}$, is calculated from the following expression:

\[
\rho_r^{k+1} = \rho_r^k - \frac{f(\rho_r^k)}{f'(\rho_r^k)}
\]
where

\[
 f' (\rho_r) = \left( R_1 + \frac{R_2}{\rho_r^2} + 2 (R_3) \rho_r - 5 (R_4) \rho_r^4 + 2 (R_5) \rho_r \right) 
 \exp \left[ -A_{11} \rho_r^2 \right] \left[ (1 + 2 A_{11} \rho_r^2) - A_{11} \rho_r^2 (1 + A_{11} \rho_r^2) \right] 
\]

**Step 4.** Steps 2–3 are repeated \( n \) times, until the error, i.e., \( \text{abs}(r_{r_k}^k - r_{r_k}^{k+1}) \), becomes smaller than a preset tolerance, e.g., \( 10^{-12} \).

**Step 5.** The correct value of \( \rho_r \) is then used to evaluate Equation 2-40 for the compressibility factor, i.e.,

\[
 z = \frac{0.27 \rho_{p_r}}{\rho_r T_{p_r}} 
\]

The proposed correlation was reported to duplicate compressibility factors from the Standing and Katz chart with an average absolute error of 0.585 percent and is applicable over the ranges:

\begin{align*}
 0.2 &< \rho_{p_r} < 30 \\
 1.0 &< T_{p_r} < 3.0
\end{align*}

**The Dranchuk-Purvis-Robinson Method**

Dranchuk, Purvis, and Robinson (1974) developed a correlation based on the Benedict-Webb-Rubin type of equation-of-state. Fitting the equation to 1,500 data points from the Standing and Katz z-factor chart optimized the eight coefficients of the proposed equations. The equation has the following form:

\[
 1 + T_1 \rho_r + T_2 \rho_r^2 + T_3 \rho_r^5 + [T_4 \rho_r^2 (1 + A_8 \rho_r^2)] 
 \exp (-A_8 \rho_r^2) - \frac{T_5}{\rho_r} = 0 
\]  \hspace{1cm} (2 - 43)

with

\[
 T_1 = \left[ A_1 + \frac{A_2}{T_{p_r}} + \frac{A_3}{T_{p_r}^3} \right] 
\]
where $r$ is defined by Equation 2-41 and the coefficients $A_1$ through $A_8$ have the following values:

$$A_1 = 0.31506237$$
$$A_2 = -1.0467099$$
$$A_3 = -0.57832720$$
$$A_4 = 0.53530771$$
$$A_5 = -0.61232032$$
$$A_6 = -0.10488813$$
$$A_7 = 0.68157001$$
$$A_8 = 0.68446549$$

The solution procedure of Equation 2-43 is similar to that of Dranchuk and Abu-Kassem. The method is valid within the following ranges of pseudo-reduced temperature and pressure:

$$1.05 \leq T_{pr} < 3.0$$
$$0.2 \leq p_{pr} \leq 3.0$$

**COMPRESSIBILITY OF NATURAL GASES**

Knowledge of the variability of fluid compressibility with pressure and temperature is essential in performing many reservoir engineering calculations. For a liquid phase, the compressibility is small and usually assumed to be constant. For a gas phase, the compressibility is neither small nor constant.

By definition, the isothermal gas compressibility is the change in volume per unit volume for a unit change in pressure or, in equation form:

$$c_g = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

(2 - 44)
where \( c_g \) = isothermal gas compressibility, 1/psi.

From the real gas equation-of-state:

\[
V = \frac{nRTz}{p}
\]

Differentiating the above equation with respect to pressure at constant temperature \( T \) gives:

\[
\left( \frac{\partial V}{\partial p} \right)_T = nRT \left[ \frac{1}{p} \left( \frac{\partial z}{\partial p} \right)_T - \frac{z}{p^2} \right]
\]

Substituting into Equation 2-44 produces the following generalized relationship:

\[
c_g = \frac{1}{p} - \frac{1}{z} \left( \frac{\partial z}{\partial p} \right)_T \quad (2 - 45)
\]

For an ideal gas, \( z = 1 \) and \( (\partial z/\partial p)_T = 0 \), therefore:

\[
c_g = \frac{1}{p} \quad (2 - 46)
\]

It should be pointed out that Equation 2-46 is useful in determining the expected order of magnitude of the isothermal gas compressibility.

Equation 2-45 can be conveniently expressed in terms of the pseudo-reduced pressure and temperature by simply replacing \( p \) with \( (p_{pc} \ p_{pr}) \), or:

\[
c_g = \frac{1}{p_{pr}} \frac{1}{p_{pc}} - \frac{1}{z} \left[ \frac{\partial z}{\partial (p_{pr} \ p_{pc})} \right]_{T_{pr}}
\]

Multiplying the above equation by \( p_{pc} \) yields:

\[
c_g \ p_{pc} = c_{pr} = \frac{1}{p_{pr}} - \frac{1}{z} \left[ \frac{\partial z}{\partial p_{pr}} \right]_{T_{pr}} \quad (2 - 47)
\]
The term $c_{pr}$ is called the isothermal pseudo-reduced compressibility and is defined by the relationship

$$c_{pr} = c_g p_{pc}$$  \hspace{1cm} (2-48)

where $c_{pr}$ = isothermal pseudo-reduced compressibility
$c_g$ = isothermal gas compressibility, psi$^{-1}$
$p_{pc}$ = pseudo-reduced pressure, psi

Values of $(\partial z/\partial p_{pr})_{T_{pr}}$ can be calculated from the slope of the $T_{pr}$ isotherm on the Standing and Katz z-factor chart.

**Example 2-10**

A hydrocarbon gas mixture has a specific gravity of 0.72. Calculate the isothermal gas compressibility coefficient at 2000 psia and 140°F by assuming:

a. An ideal gas behavior
b. A real gas behavior

**Solution**

a. Assuming an ideal gas behavior, determine $c_g$ by applying Equation 2-45:

$$c_g = \frac{1}{2000} = 500 \times 10^{-6} \text{ psi}^{-1}$$

b. Assuming a real gas behavior

*Step 1.* Calculate $T_{pc}$ and $p_{pc}$ by applying Equations 2-17 and 2-18

$$T_{pc} = 168 + 325 (0.72) - 12.5 (0.72)^2 = 395.5 \text{ °R}$$

$$P_{pc} = 677 + 15 (0.72) - 37.5 (0.72)^2 = 668.4 \text{ psia}$$

*Step 2.* Compute $p_{pr}$ and $T_{pr}$ from Equations 2-11 and 2-12.

$$p_{pr} = \frac{2000}{668.4} = 2.99$$

$$T_{pr} = \frac{600}{395.5} = 1.52$$
Step 3. Determine the $z$-factor from Figure 2-1:

$$z = 0.78$$

Step 4. Calculate the slope $\left[ \frac{\partial z}{\partial p_{pr}} \right]_{T_{pr}} = 1.52$:

$$\left[ \frac{\partial z}{\partial p_{pr}} \right]_{T_{pr}} = -0.022$$

Step 5. Solve for $c_{pr}$ by applying Equation 2-47:

$$c_{pr} = \frac{1}{2.99} - \frac{1}{0.78} \times [-0.022] = 0.3627$$

Step 6. Calculate $c_g$ from Equation 2-48:

$$c_g = \frac{0.327}{668.4} = 543 \times 10^{-6} \text{ psi}^{-1}$$

Trube (1957) presented graphs from which the isothermal compressibility of natural gases may be obtained. The graphs, as shown in Figures 2-3 and 2-4 give the isothermal pseudo-reduced compressibility as a function of pseudo-reduced pressure and temperature.

Example 2-11

Using Trube’s generalized charts, rework Example 2-10.

Solution

Step 1. From Figure 2-3, find $c_{pr}$:

$$c_{pr} = 0.36$$

Step 2. Solve for $c_g$ by applying Equation 2-49:

$$c_g = \frac{0.36}{668.4} = 539 \times 10^{-6} \text{ psi}^{-1}$$

Matter, Brar, and Aziz (1975) presented an analytical technique for calculating the isothermal gas compressibility. The authors expressed $c_{pr}$
as a function of $\frac{\partial p}{\partial \rho}$, rather than $\frac{\partial p}{\partial \rho_{pr}}$.

Equation 2-41 is differentiated with respect to $\rho_{pr}$ to give:

$$
\left[ \frac{\partial z}{\partial \rho_{pr}} \right] = 0.27 \frac{z}{T_{pr}} \left[ \frac{(\partial z/\partial \rho_{r})_{T_{pr}}}{1 + \frac{\rho_{r}}{z} (\partial z/\partial \rho_{r})_{T_{pr}}} \right]
$$

Equation 2-49 may be substituted into Equation 2-47 to express the pseudo-reduced compressibility as:
Figure 2-4. Trube's pseudo-reduced compressibility for natural gases. (Permission to publish by the Society of Petroleum Engineers of AIME. Copyright SPE-AIME.)

\[ c_{pr} = \frac{1}{p_{pr}} - \frac{0.27}{z^2 T_{pr}} \left[ \frac{\partial z/\partial \rho_r}{T_{pr}} \right] \left[ 1 + \frac{\rho_r}{z} \left( \frac{\partial z/\partial \rho_r}{T_{pr}} \right) \right] \] (2 - 50)
where $\rho_r = \text{pseudo-reduced gas density.}$

The partial derivative appearing in Equation 2-50 is obtained from Equation 2-43 to give:

$$
\left[ \frac{\partial z}{\partial \rho_r} \right]_{T_{pr}} = T_1 + 2T_2 \rho_r + 5T_3 \rho_r^4 + 2T_4 \rho_r^8 (1 + A_8 \rho_r^2 - A_8 \rho_r^4) \\
\times \exp (-A_8 \rho_r^2)
$$

(2-51)

where the coefficients $T_1$ through $T_4$ and $A_1$ through $A_8$ are defined previously by Equation 2-43.

**GAS FORMATION VOLUME FACTOR**

The gas formation volume factor is used to relate the volume of gas, as measured at reservoir conditions, to the volume of the gas as measured at standard conditions, i.e., 60°F and 14.7 psia. This gas property is then defined as the actual volume occupied by a certain amount of gas at a specified pressure and temperature, divided by the volume occupied by the same amount of gas at standard conditions. In an equation form, the relationship is expressed as

$$
B_g = \frac{V_{p,T}}{V_{sc}}
$$

(2-52)

where $B_g = \text{gas formation volume factor, ft}^3/\text{scf}$

$V_{p,T} = \text{volume of gas at pressure } p \text{ and temperature, } T, \text{ ft}^3$

$V_{sc} = \text{volume of gas at standard conditions, scf}$

Applying the real gas equation-of-state, i.e., Equation 2-11, and substituting for the volume $V$, gives:

$$
B_g = \frac{z_n RT}{p} \frac{1}{z_{sc} \frac{n R}{T_{sc}} \frac{p_{sc}}{T_{sc}}}
$$

$$
= \frac{p_{sc}}{p} \frac{z_{sc} T_{sc}}{T_{sc}}
$$

(2-53)

where $z_{sc} = \text{z-factor at standard conditions} = 1.0$

$p_{sc}, T_{sc} = \text{standard pressure and temperature}$

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Assuming that the standard conditions are represented by \( p_{sc} = 14.7 \) psia and \( T_{sc} = 520 \), the above expression can be reduced to the following relationship:

\[
B_g = 0.02827 \frac{zT}{p} \tag{2-53}
\]

where \( B_g \) = gas formation volume factor, \( \text{ft}^3/\text{scf} \)
\( z \) = gas compressibility factor
\( T \) = temperature, °R

In other field units, the gas formation volume factor can be expressed in \( \text{bbl/scf} \), to give:

\[
B_g = 0.005035 \frac{zT}{p} \tag{2-54}
\]

The reciprocal of the gas formation volume factor is called the gas expansion factor and is designated by the symbol \( E_g \), or:

\[
E_g = 35.37 \frac{p}{zT}, \text{scf/ft}^3 \tag{2-55}
\]

In other units:

\[
E_g = 198.6 \frac{p}{zT}, \text{scf/bbl} \tag{2-56}
\]

**Example 2-12**

A gas well is producing at a rate of 15,000 \( \text{ft}^3/\text{day} \) from a gas reservoir at an average pressure of 2,000 psia and a temperature of 120°F. The specific gravity is 0.72. Calculate the gas flow rate in \( \text{scf/day} \).

**Solution**

*Step 1.* Calculate the pseudo-critical properties from Equations 2-17 and 2-18, to give:

\[
T_{pc} = 395.5 \text{ °R} \quad p_{pc} = 668.4 \text{ psia}
\]
Step 2. Calculate the $p_{pr}$ and $T_{pr}$:

$$p_{pr} = \frac{2000}{668.4} = 2.99$$

$$T_{pr} = \frac{600}{395.5} = 1.52$$

Step 3. Determine the $z$-factor from Figure 2-1:

$z = 0.78$

Step 4. Calculate the gas expansion factor from Equation 2-55:

$$E_g = 35.37 \times \frac{2000}{(0.78)(600)} = 151.15 \text{ scf/ft}^3$$

Step 5. Calculate the gas flow rate in scf/day by multiplying the gas flow rate (in ft$^3$/day) by the gas expansion factor $E_g$ as expressed in scf/ft$^3$:

Gas flow rate = (151.15)(15,000) = 2.267 MMscf/day

**GAS VISCOSITY**

The viscosity of a fluid is a measure of the internal fluid friction (resistance) to flow. If the friction between layers of the fluid is small, i.e., low viscosity, an applied shearing force will result in a large velocity gradient. As the viscosity increases, each fluid layer exerts a larger frictional drag on the adjacent layers and velocity gradient decreases.

The viscosity of a fluid is generally defined as the ratio of the shear force per unit area to the local velocity gradient. Viscosities are expressed in terms of poises, centipoise, or micropoises. One poise equals a viscosity of 1 dyne-sec/cm$^2$ and can be converted to other field units by the following relationships:

1 poise = 100 centipoises

= $1 \times 10^6$ micropoises

= $6.72 \times 10^{-2}$ lb mass/ft-sec

= $2.09 \times 10^{-3}$ lb-sec/ft$^2$
The gas viscosity is not commonly measured in the laboratory because it can be estimated precisely from empirical correlations. Like all intensive properties, viscosity of a natural gas is completely described by the following function:

$$\mu_g = (p, T, y_i)$$

where $\mu_g$ = the viscosity of the gas phase. The above relationship simply states that the viscosity is a function of pressure, temperature, and composition. Many of the widely used gas viscosity correlations may be viewed as modifications of that expression.

**METHODS OF CALCULATING THE VISCOSITY OF NATURAL GASES**

Two popular methods that are commonly used in the petroleum industry are the:

- Carr-Kobayashi-Burrows Correlation Method
- Lee-Gonzalez-Eakin Method

**The Carr-Kobayashi-Burrows Correlation Method**

Carr, Kobayashi, and Burrows (1954) developed graphical correlations for estimating the viscosity of natural gas as a function of temperature, pressure, and gas gravity. The computational procedure of applying the proposed correlations is summarized in the following steps:

**Step 1.** Calculate the pseudo-critical pressure, pseudo-critical temperature, and apparent molecular weight from the specific gravity or the composition of the natural gas. Corrections to these pseudo-critical properties for the presence of the nonhydrocarbon gases (CO₂, N₂, and H₂S) should be made if they are present in concentrations greater than 5 mole percent.

**Step 2.** Obtain the viscosity of the natural gas at one atmosphere and the temperature of interest from Figure 2-5. This viscosity, as denoted by $\mu_1$, must be corrected for the presence of nonhydrocarbon components by using the inserts of Figure 2-5. The nonhydrocarbon fractions tend to increase the viscosity of the gas phase. The effect of nonhydrocarbon components on the viscosity of the nat-
Figure 2-5. Carr’s atmospheric gas viscosity correlation. (Permission to publish by the Society of Petroleum Engineers of AIME. Copyright SPE-AIME.)
ural gas can be expressed mathematically by the following relationships:

\[ \mu_1 = (\mu_1)_{\text{uncorrected}} + (\Delta \mu)_{N_2} + (\Delta \mu)_{CO_2} + (\Delta \mu)_{H_2S} \]  

(2-57)

where \( \mu_1 \) = “corrected” gas viscosity at one atmospheric pressure and reservoir temperature, cp

- \( (\Delta \mu)_{N_2} \) = viscosity corrections due to the presence of \( N_2 \)
- \( (\Delta \mu)_{CO_2} \) = viscosity corrections due to the presence of \( CO_2 \)
- \( (\Delta \mu)_{H_2S} \) = viscosity corrections due to the presence of \( H_2S \)

\( (\mu_1)_{\text{uncorrected}} \) = uncorrected gas viscosity, cp

**Step 3.** Calculate the pseudo-reduced pressure and temperature.

**Step 4.** From the pseudo-reduced temperature and pressure, obtain the viscosity ratio \( \mu_g/\mu_1 \) from Figure 2-6. The term \( \mu_g \) represents the viscosity of the gas at the required conditions.

**Step 5.** The gas viscosity, \( \mu_g \), at the pressure and temperature of interest is calculated by multiplying the viscosity at one atmosphere and system temperature, \( \mu_1 \), by the viscosity ratio.

The following examples illustrate the use of the proposed graphical correlations:

**Example 2-13**

Using the data given in Example 2-12, calculate the viscosity of the gas.

**Solution**

**Step 1.** Calculate the apparent molecular weight of the gas:

\[ M_a = (0.72) (28.96) = 20.85 \]

**Step 2.** Determine the viscosity of the gas at 1 atm and 140°F from Figure 2-5:

\[ \mu_1 = 0.0113 \]
Figure 2-6. Carr’s viscosity ratio correlation. (Permission to publish by the Society of Petroleum Engineers of AIME. Copyright SPE-AIME.)
Step 3. Calculate $p_{pr}$ and $T_{pr}$:

\[
p_{pr} = 2.99
\]
\[
T_{pr} = 1.52
\]

Step 4. Determine the viscosity rates from Figure 2-6:

\[
\frac{\mu_g}{\mu_1} = 1.5
\]

Step 5. Solve for the viscosity of the natural gas:

\[
\frac{\mu_g}{\mu_1} = \frac{(1.5)(0.0113)}{(1.5)} = 0.01695 \text{ cp}
\]

Standing (1977) proposed a convenient mathematical expression for calculating the viscosity of the natural gas at atmospheric pressure and reservoir temperature, $\mu_1$. Standing also presented equations for describing the effects of $N_2$, $CO_2$, and $H_2S$ on $\mu_1$. The proposed relationships are:

\[
\mu_1 = (\mu_1)_{uncorrected} + (\Delta \mu)_{CO_2} + (\Delta \mu)_{H_2S} + (\Delta \mu)_{N_2}
\]

(2 - 58)

where:

\[
(\mu_1)_{uncorrected} = [1.709 \times 10^{-5} - 2.062 \times 10^{-6} \gamma_g] (T - 460) + 8.118 \times 10^{-3} - 6.15 \times 10^{-3} \log (\gamma_g)
\]

(2 - 59)

\[
(\Delta \mu)_{CO_2} = y_{CO_2} [0.08 \times 10^{-3} \log (\gamma_g) + (6.24 \times 10^{-3})]
\]

(2 - 60)

\[
(\Delta \mu)_{H_2S} = y_{H_2S} [8.49 \times 10^{-3} \log (\gamma_g) + 9.59 \times 10^{-3}]
\]

(2 - 61)

where

$\mu_1$ = viscosity of the gas at atmospheric pressure and reservoir temperature, cp

$T$ = reservoir temperature, ºR

$\gamma_g$ = gas gravity

$y_{N_2}$, $y_{CO_2}$, $y_{H_2S}$ = mole fraction of $N_2$, $CO_2$, and $H_2S$, respectively

Dempsey (1965) expressed the viscosity ratio $\frac{\mu_g}{\mu_1}$ by the following relationship:
where \( T_{pr} \) = pseudo-reduced temperature of the gas mixture, °R  
\( p_{pr} \) = pseudo-reduced pressure of the gas mixture, psia

\[ a_0 \ldots a_{17} = \text{coefficients of the equations are given below:} \]

\[ a_0 = -2.46211820 \quad a_8 = -7.93385648 \times 10^{-1} \]
\[ a_1 = 2.970547414 \quad a_9 = 1.39643306 \]
\[ a_2 = -2.86264054 \times 10^{-1} \quad a_{10} = -1.49144925 \times 10^{-1} \]
\[ a_3 = 8.05420522 \times 10^{-3} \quad a_{11} = 4.41015512 \times 10^{-3} \]
\[ a_4 = 2.80860949 \quad a_{12} = 8.39387178 \times 10^{-2} \]
\[ a_5 = -3.49803305 \quad a_{13} = -1.86408848 \times 10^{-1} \]
\[ a_6 = 3.60373020 \times 10^{-1} \quad a_{14} = 2.03367881 \times 10^{-2} \]
\[ a_7 = -1.044324 \times 10^{-2} \quad a_{15} = -6.09579263 \times 10^{-4} \]

**The Lee-Gonzalez-Eakin Method**

Lee, Gonzalez, and Eakin (1966) presented a semi-empirical relationship for calculating the viscosity of natural gases. The authors expressed the gas viscosity in terms of the reservoir temperature, gas density, and the molecular weight of the gas. Their proposed equation is given by:

\[ \mu_g = 10^{-4} \ K \exp \left[ X \left( \frac{\rho_g}{62.4} \right)^Y \right] \quad (2 \ - 63) \]

where

\[ K = \frac{(9.4 + 0.02 \ M_a) \ T^{1.5}}{209 + 19 \ M_a + T} \quad (2 \ - 64) \]

\[ X = 3.5 + \frac{986}{T} + 0.01 \ M_a \quad (2 \ - 65) \]

\[ Y = 2.4 - 0.2 \ X \quad (2 \ - 66) \]
The proposed correlation can predict viscosity values with a standard deviation of 2.7% and a maximum deviation of 8.99%. The correlation is less accurate for gases with higher specific gravities. The authors pointed out that the method cannot be used for sour gases.

Example 2-14

Rework Example 2-13 and calculate the gas viscosity by using the Lee-Gonzalez-Eakin method.

Step 1. Calculate the gas density from Equation 2-16:

\[
\rho_g = \frac{(2000)(20.85)}{(10.73)(600)(0.78)} = 8.3 \text{ lb/ft}^3
\]

Step 2. Solve for the parameters K, X, and Y by using Equations 2-64, 2-65, and 2-66, respectively:

\[
K = \frac{[9.4 + 0.02(20.85)][(600)^{1.5}]}{209 + 19(20.85) + 600} = 119.72
\]

\[
X = 3.5 + \frac{986}{600} + 0.01(20.85) = 5.35
\]

\[
Y = 2.4 - 0.2(5.35) = 1.33
\]

Step 3. Calculate the viscosity from Equation 2-63:

\[
\mu_g = 10^{-4}(119.72)\exp\left[5.35 \left(\frac{8.3}{62.4}\right)^{1.33}\right] = 0.0173 \text{ cp}
\]

**PROPERTIES OF CRUDE OIL SYSTEMS**

Petroleum (an equivalent term is crude oil) is a complex mixture consisting predominantly of hydrocarbons and containing sulfur, nitrogen, oxygen, and helium as minor constituents. The physical and chemical properties of crude oils vary considerably and are dependent on the con-
centration of the various types of hydrocarbons and minor constituents present.

An accurate description of physical properties of crude oils is of a considerable importance in the fields of both applied and theoretical science and especially in the solution of petroleum reservoir engineering problems. Physical properties of primary interest in petroleum engineering studies include:

- Fluid gravity
- Specific gravity of the solution gas
- Gas solubility
- Bubble-point pressure
- Oil formation volume factor
- Isothermal compressibility coefficient of undersaturated crude oils
- Oil density
- Total formation volume factor
- Crude oil viscosity
- Surface tension

Data on most of these fluid properties are usually determined by laboratory experiments performed on samples of actual reservoir fluids. In the absence of experimentally measured properties of crude oils, it is necessary for the petroleum engineer to determine the properties from empirically derived correlations.

**Crude Oil Gravity**

The crude oil density is defined as the mass of a unit volume of the crude at a specified pressure and temperature. It is usually expressed in pounds per cubic foot. The specific gravity of a crude oil is defined as the ratio of the density of the oil to that of water. Both densities are measured at 60°F and atmospheric pressure:

\[
\gamma_o = \frac{\rho_o}{\rho_w}
\]  

where  
\( \gamma_o \) = specific gravity of the oil  
\( \rho_o \) = density of the crude oil, lb/ft\(^3\)  
\( \rho_w \) = density of the water, lb/ft\(^3\)
It should be pointed out that the liquid specific gravity is dimensionless, but traditionally is given the units 60°/60° to emphasize the fact that both densities are measured at standard conditions. The density of the water is approximately 62.4 lb/ft³, or:

\[ \gamma_o = \frac{\rho_o}{62.4}, \text{60°/60°} \]

Although the density and specific gravity are used extensively in the petroleum industry, the API gravity is the preferred gravity scale. This gravity scale is precisely related to the specific gravity by the following expression:

\[ ^\circ\text{API} = \frac{141.5}{\gamma_o} - 131.5 \quad (2 - 68) \]

The API gravities of crude oils usually range from 47° API for the lighter crude oils to 10° API for the heavier asphaltic crude oils.

**Example 2-15**

Calculate the specific gravity and the API gravity of a crude oil system with a measured density of 53 lb/ft³ at standard conditions.

**Solution**

*Step 1.* Calculate the specific gravity from Equation 2-67:

\[ \gamma_o = \frac{53}{62.4} = 0.849 \]

*Step 2.* Solve for the API gravity:

\[ \text{API} = \frac{141.5}{0.849} - 131.5 = 35.2° \text{ API} \]

**Specific Gravity of the Solution Gas**

The specific gravity of the solution gas \( \gamma_g \) is described by the weighted average of the specific gravities of the separated gas from each separator.
This weighted-average approach is based on the separator gas-oil ratio, or:

\[ \gamma_g = \frac{\sum_{i=1}^{n} (R_{sep})_i (\gamma_{sep})_i + R_{st} \gamma_{st}}{\sum_{i=1}^{n} (R_{sep})_i + R_{st}} \]  

(2 - 69)

where  
- \( n \) = number of separators  
- \( R_{sep} \) = separator gas-oil ratio, scf/STB  
- \( \gamma_{sep} \) = separator gas gravity  
- \( R_{st} \) = gas-oil ratio from the stock tank, scf/STB  
- \( \gamma_{st} \) = gas gravity from the stock tank

**Example 2-16**

Separator tests were conducted on a crude oil sample. Results of the test in terms of the separator gas-oil ration and specific gravity of the separated gas are given below:

<table>
<thead>
<tr>
<th>Separator #</th>
<th>Pressure psig</th>
<th>Temperature °F</th>
<th>Gas-Oil Ratio scf/STB</th>
<th>Gas Specific Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>660</td>
<td>150</td>
<td>724</td>
<td>0.743</td>
</tr>
<tr>
<td>Intermediate</td>
<td>75</td>
<td>110</td>
<td>202</td>
<td>0.956</td>
</tr>
<tr>
<td>Stock tank</td>
<td>0</td>
<td>60</td>
<td>58</td>
<td>1.296</td>
</tr>
</tbody>
</table>

Calculate the specific gravity of the separated gas.

**Solution**

Estimate the specific gravity of the solution by using Equation 2-69:

\[ \gamma_g = \frac{(724)(0.743) + (202)(0.956) + (58)(1.296)}{724 + 202 + 58} = 0.819 \]

**Gas Solubility**

The gas solubility \( R_s \) is defined as the number of standard cubic feet of gas which will dissolve in one stock-tank barrel of crude oil at certain
pressure and temperature. The solubility of a natural gas in a crude oil is a strong function of the pressure, temperature, API gravity, and gas gravity.

For a particular gas and crude oil to exist at a constant temperature, the solubility increases with pressure until the saturation pressure is reached. At the saturation pressure (bubble-point pressure) all the available gases are dissolved in the oil and the gas solubility reaches its maximum value. Rather than measuring the amount of gas that will dissolve in a given stock-tank crude oil as the pressure is increased, it is customary to determine the amount of gas that will come out of a sample of reservoir crude oil as pressure decreases.

A typical gas solubility curve, as a function of pressure for an undersaturated crude oil, is shown in Figure 2-7. As the pressure is reduced from the initial reservoir pressure $p_i$, to the bubble-point pressure $p_b$, no gas evolves from the oil and consequently the gas solubility remains constant at its maximum value of $R_s$. Below the bubble-point pressure, the solution gas is liberated and the value of $R_s$ decreases with pressure. The following five empirical correlations for estimating the gas solubility are given below:

- Standing’s correlation
- The Vasquez-Beggs correlation
- Glaso’s correlation
- Marhoun’s correlation
- The Petrosky-Farshad correlation

**Standing’s Correlation**

Standing (1947) proposed a graphical correlation for determining the gas solubility as a function of pressure, gas specific gravity, API gravity, and system temperature. The correlation was developed from a total of 105 experimentally determined data points on 22 hydrocarbon mixtures from California crude oils and natural gases. The proposed correlation has an average error of 4.8%. Standing (1981) expressed his proposed graphical correlation in the following more convenient mathematical form:

$$ R_s = \gamma_g \left[ \left( \frac{p}{18.2} + 1.4 \right) 10^4 \right]^{1.2048} \quad (2 - 70) $$
with

\[ x = 0.0125 \text{ API} - 0.00091(T - 460) \]

where \( T \) = temperature, °R
\( p \) = system pressure, psia
\( \gamma_g \) = solution gas specific gravity

It should be noted that Standing’s equation is valid for applications at and below the bubble-point pressure of the crude oil.

**Example 2-17**

The following experimental PVT data on six different crude oil systems are available. Results are based on two-stage surface separation.
where $T$ = reservoir temperature, °F
$p_b$ = bubble-point pressure, psig
$B_o$ = oil formation volume factor, bbl/STB
$p_{sep}$ = separator pressure, psig
$T_{sep}$ = separator temperature, °F
$c_o$ = isothermal compressibility coefficient of the oil at a specified pressure, psi$^{-1}$

Using Standing’s correlation, estimate the gas solubility at the bubble-point pressure and compare with the experimental value in terms of the absolute average error (AAE).

**Solution**

Apply Equation 2-70 to determine the gas solubility. Results of the calculations are given in the following tabulated form:

<table>
<thead>
<tr>
<th>Oil #</th>
<th>$T$</th>
<th>$p_b$</th>
<th>$R_s$</th>
<th>$B_o$</th>
<th>$c_o$ at $p/p_b$</th>
<th>$p_{sep}$</th>
<th>$T_{sep}$</th>
<th>API</th>
<th>$\gamma_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>2377</td>
<td>751</td>
<td>1.528</td>
<td>38.13</td>
<td>150</td>
<td>60</td>
<td>47.1</td>
<td>0.851</td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>2620</td>
<td>768</td>
<td>1.474</td>
<td>40.95</td>
<td>100</td>
<td>75</td>
<td>40.7</td>
<td>0.855</td>
</tr>
<tr>
<td>3</td>
<td>260</td>
<td>2051</td>
<td>693</td>
<td>1.529</td>
<td>37.37</td>
<td>2526</td>
<td>100</td>
<td>48.6</td>
<td>0.911</td>
</tr>
<tr>
<td>4</td>
<td>237</td>
<td>2884</td>
<td>968</td>
<td>1.619</td>
<td>38.92</td>
<td>2942</td>
<td>60</td>
<td>40.5</td>
<td>0.898</td>
</tr>
<tr>
<td>5</td>
<td>218</td>
<td>3045</td>
<td>943</td>
<td>1.570</td>
<td>37.70</td>
<td>3273</td>
<td>200</td>
<td>44.2</td>
<td>0.781</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>4239</td>
<td>807</td>
<td>1.385</td>
<td>46.79</td>
<td>4370</td>
<td>85</td>
<td>27.3</td>
<td>0.848</td>
</tr>
</tbody>
</table>

$\text{AAE} = 10.1\%$

**The Vasquez-Beggs Correlation**

Vasquez and Beggs (1980) presented an improved empirical correlation for estimating $R_s$. The correlation was obtained by regression analy-
sis using 5,008 measured gas solubility data points. Based on oil gravity, the measured data were divided into two groups. This division was made at a value of oil gravity of 30°API. The proposed equation has the following form:

\[ R_s = C_1 \gamma_{gs} p^{C_2} \exp \left[ C_3 \left( \frac{\text{API}}{T} \right) \right] \]  

(2-71)

Values for the coefficients are as follows:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>API ≤ 30</th>
<th>API &gt; 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>0.0362</td>
<td>0.0178</td>
</tr>
<tr>
<td>C_2</td>
<td>1.0937</td>
<td>1.1870</td>
</tr>
<tr>
<td>C_3</td>
<td>25.7240</td>
<td>23.931</td>
</tr>
</tbody>
</table>

Realizing that the value of the specific gravity of the gas depends on the conditions under which it is separated from the oil, Vasquez and Beggs proposed that the value of the gas specific gravity as obtained from a separator pressure of 100 psig be used in the above equation. This reference pressure was chosen because it represents the average field separator conditions. The authors proposed the following relationship for adjustment of the gas gravity \( \gamma_g \) to the reference separator pressure:

\[ \gamma_{gs} = \gamma_g \left[ 1 + 5.912 \times 10^{-5} \left( \text{API} \right) \left( T_{\text{sep}} - 460 \right) \log \left( \frac{p_{\text{sep}}}{114.7} \right) \right] \]  

(2-72)

where \( \gamma_{gs} \) = gas gravity at the reference separator pressure 
\( \gamma_g \) = gas gravity at the actual separator conditions of \( p_{\text{sep}} \) and \( T_{\text{sep}} \) 
\( p_{\text{sep}} \) = actual separator pressure, psia 
\( T_{\text{sep}} \) = actual separator temperature, °R

The gas gravity used to develop all the correlations reported by the authors was that which would result from a two-stage separation. The first-stage pressure was chosen as 100 psig and the second stage was the stock tank. If the separator conditions are unknown, the unadjusted gas gravity may be used in Equation 2-71.

An independent evaluation of the above correlation by Sutton and Farashad (1984) shows that the correlation is capable of predicting gas solubilities with an average absolute error of 12.7%.
Example 2-18

Using the PVT of the six crude oil systems of Example 2-17, solve for the gas solubility.

Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>$\gamma_g$ From Equation 2-72</th>
<th>Predicted $R_s$ Equation 2-71</th>
<th>Measured $R_s$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8731</td>
<td>779</td>
<td>751</td>
<td>3.76</td>
</tr>
<tr>
<td>2</td>
<td>0.855</td>
<td>733</td>
<td>768</td>
<td>-4.58</td>
</tr>
<tr>
<td>3</td>
<td>0.911</td>
<td>702</td>
<td>693</td>
<td>1.36</td>
</tr>
<tr>
<td>4</td>
<td>0.850</td>
<td>820</td>
<td>968</td>
<td>15.2</td>
</tr>
<tr>
<td>5</td>
<td>0.814</td>
<td>947</td>
<td>943</td>
<td>0.43</td>
</tr>
<tr>
<td>6</td>
<td>0.834</td>
<td>841</td>
<td>807</td>
<td>4.30</td>
</tr>
</tbody>
</table>

$\text{AAE} = 4.9\%$

Glaso’s Correlation

Glaso (1980) proposed a correlation for estimating the gas solubility as a function of the API gravity, pressure, temperature, and gas specific gravity. The correlation was developed from studying 45 North Sea crude oil samples. Glaso reported an average error of 1.28% with a standard deviation of 6.98%. The proposed relationship has the following form:

$$R_s = \gamma_g \left[ \left( \frac{\text{API}^{0.989}}{(T - 460)^{0.177}} \right) (p_t^b) \right]^{1.2255}$$

(2 - 73)

where $p_t^b$ is a correlating number and is defined by the following expression:

$$p_t^b = 10^x$$

with

$$x = 2.8869 - [14.1811 - 3.3093 \log (p)]^{0.5}$$

Example 2-19

Rework Example 2-17 and solve for the gas solubility by using Glaso’s correlation.
Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>x</th>
<th>p_s</th>
<th>Predicted R_s</th>
<th>Measured R_s</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.155</td>
<td>14.286</td>
<td>737</td>
<td>751</td>
<td>–1.84</td>
</tr>
<tr>
<td>2</td>
<td>1.196</td>
<td>15.687</td>
<td>714</td>
<td>768</td>
<td>–6.92</td>
</tr>
<tr>
<td>3</td>
<td>1.095</td>
<td>12.450</td>
<td>686</td>
<td>693</td>
<td>–0.90</td>
</tr>
<tr>
<td>4</td>
<td>1.237</td>
<td>17.243</td>
<td>843</td>
<td>968</td>
<td>–12.92</td>
</tr>
<tr>
<td>5</td>
<td>1.260</td>
<td>18.210</td>
<td>868</td>
<td>943</td>
<td>–7.95</td>
</tr>
<tr>
<td>6</td>
<td>1.413</td>
<td>25.883</td>
<td>842</td>
<td>807</td>
<td>4.34</td>
</tr>
</tbody>
</table>

AAE = 5.8%

Marhoun’s Correlation

Marhoun (1988) developed an expression for estimating the saturation pressure of the Middle Eastern crude oil systems. The correlation originates from 160 experimental saturation pressure data. The proposed correlation can be rearranged and solved for the gas solubility to give:

\[ R_s = \left[ a \gamma_g^b \gamma_o^c T^d p \right]^e \]  

(2-74)

where

- \( \gamma_g \) = gas specific gravity
- \( \gamma_o \) = stock-tank oil gravity
- \( T \) = temperature, °R
- \( a, b, c, d, e \) = coefficients of the above equation having these values:
  - \( a = 185.843208 \)
  - \( b = 1.877840 \)
  - \( c = -3.1437 \)
  - \( d = -1.32657 \)
  - \( e = 1.398441 \)

Example 2-20

Resolve Example 2-17 by using Marhoun’s correlation.
### Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Predicted $R_s$ Equation 2-74</th>
<th>Measured $R_s$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>740</td>
<td>751</td>
<td>-1.43</td>
</tr>
<tr>
<td>2</td>
<td>792</td>
<td>768</td>
<td>3.09</td>
</tr>
<tr>
<td>3</td>
<td>729</td>
<td>693</td>
<td>5.21</td>
</tr>
<tr>
<td>4</td>
<td>1041</td>
<td>968</td>
<td>7.55</td>
</tr>
<tr>
<td>5</td>
<td>845</td>
<td>943</td>
<td>-10.37</td>
</tr>
<tr>
<td>6</td>
<td>1186</td>
<td>807</td>
<td>47.03</td>
</tr>
</tbody>
</table>

AAE = 12.4%

### The Petrosky-Farshad Correlation

Petrosky and Farshad (1993) used a nonlinear multiple regression software to develop a gas solubility correlation. The authors constructed a PVT database from 81 laboratory analyses from the Gulf of Mexico crude oil system. Petrosky and Farshad proposed the following expression:

$$R_s = \left[ \frac{P}{112.727} + 12.340 \right] g^{0.8439} 10^{x} 1.73184$$

(2-75)

with

$$x = 7.916 \times 10^{-4} (\text{API})^{1.5410} - 4.561 \times 10^{-5} (T - 460)^{1.3911}$$

where $P$ = pressure, psia

$T$ = temperature, °R

### Example 2-21

Test the predictive capability of the Petrosky and Farshad equation by resolving Example 2-17.
Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>x</th>
<th>Predicted $R_s$ Equation 2-75</th>
<th>Measured $R_s$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2008</td>
<td>772</td>
<td>751</td>
<td>2.86</td>
</tr>
<tr>
<td>2</td>
<td>0.1566</td>
<td>726</td>
<td>768</td>
<td>-5.46</td>
</tr>
<tr>
<td>3</td>
<td>0.2101</td>
<td>758</td>
<td>693</td>
<td>9.32</td>
</tr>
<tr>
<td>4</td>
<td>0.1579</td>
<td>875</td>
<td>968</td>
<td>-9.57</td>
</tr>
<tr>
<td>5</td>
<td>0.1900</td>
<td>865</td>
<td>943</td>
<td>-8.28</td>
</tr>
<tr>
<td>6</td>
<td>0.0667</td>
<td>900</td>
<td>807</td>
<td>11.57</td>
</tr>
</tbody>
</table>

AAE = 7.84%

The gas solubility can also be calculated rigorously from the experimental measured PVT data at the specified pressure and temperature. The following expression relates the gas solubility $R_s$ to oil density, specific gravity of the oil, gas gravity, and the oil formation volume factor:

$$R_s = \frac{B_o \rho_o - 62.4 \gamma_o}{0.0136 \gamma_g} \quad (2 \cdot 76)$$

where
- $\rho_o =$ oil density, lb/ft$^3$
- $B_o =$ oil formation volume factor, bbl/STB
- $\gamma_o =$ specific gravity of the stock-tank oil
- $\gamma_g =$ specific gravity of the solution gas

McCain (1991) pointed out that the weight average of separator and stock-tank gas specific gravities should be used for $\gamma_g$. The error in calculating $R_s$ by using the above equation will depend only on the accuracy of the available PVT data.

Example 2-22

Using the data of Example 2-17, estimate $R_s$ by applying Equation 2-76.
Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Predicted $R_S$ Equation 2-76</th>
<th>Measured $R_S$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>762</td>
<td>751</td>
<td>1.53</td>
</tr>
<tr>
<td>2</td>
<td>781</td>
<td>768</td>
<td>1.73</td>
</tr>
<tr>
<td>3</td>
<td>655</td>
<td>693</td>
<td>-5.51</td>
</tr>
<tr>
<td>4</td>
<td>956</td>
<td>968</td>
<td>-1.23</td>
</tr>
<tr>
<td>5</td>
<td>841</td>
<td>943</td>
<td>-10.79</td>
</tr>
<tr>
<td>6</td>
<td>798</td>
<td>807</td>
<td>-1.13</td>
</tr>
</tbody>
</table>

$\text{AAE} = 3.65\%$ 

Bubble-Point Pressure

The bubble-point pressure $p_b$ of a hydrocarbon system is defined as the highest pressure at which a bubble of gas is first liberated from the oil. This important property can be measured experimentally for a crude oil system by conducting a constant-composition expansion test.

In the absence of the experimentally measured bubble-point pressure, it is necessary for the engineer to make an estimate of this crude oil property from the readily available measured producing parameters. Several graphical and mathematical correlations for determining $p_b$ have been proposed during the last four decades. These correlations are essentially based on the assumption that the bubble-point pressure is a strong function of gas solubility $R_S$, gas gravity $g_g$, oil gravity API, and temperature $T$, or:

$$p_b = f\left(R_S, g_g, \text{API}, T\right)$$

Several ways of combining the above parameters in a graphical form or a mathematical expression are proposed by numerous authors, including:

- Standing
- Vasquez and Beggs
- Glaso
- Marhoun
- Petrosky and Farshad

The empirical correlations for estimating the bubble-point pressure proposed by the above-listed authors are given below.
Standing’s Correlation

Based on 105 experimentally measured bubble-point pressures on 22 hydrocarbon systems from California oil fields, Standing (1947) proposed a graphical correlation for determining the bubble-point pressure of crude oil systems. The correlating parameters in the proposed correlation are the gas solubility \( R_s \), gas gravity \( \gamma_g \), oil API gravity, and the system temperature. The reported average error is 4.8%.

In a mathematical form, Standing (1981) expressed the graphical correlation by the following expression:

\[
p_b = 18.2 \left( \frac{R_s}{\gamma_g} \right)^{0.83} (10)^a - 1.4
\]

with

\[
a = 0.00091 (T - 460) - 0.0125 \text{ (API)}
\]

where \( p_b \) = bubble-point pressure, psia
\( T = \) system temperature, °R

Standing’s correlation should be used with caution if nonhydrocarbon components are known to be present in the system.

Example 2-23

The experimental data given in Example 2-17 are repeated here for convenience.

<table>
<thead>
<tr>
<th>Oil #</th>
<th>T</th>
<th>( p_b )</th>
<th>( R_s )</th>
<th>( B_o )</th>
<th>( p_o )</th>
<th>( c_g )</th>
<th>af</th>
<th>( p &gt; p_b )</th>
<th>( p_{sep} )</th>
<th>( T_{sep} )</th>
<th>API</th>
<th>( \gamma_g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>2377</td>
<td>751</td>
<td>1.528</td>
<td>38.13</td>
<td>22.14 \times 10^{-6} at 2689</td>
<td>150</td>
<td>60</td>
<td>47.1</td>
<td>0.851</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>2620</td>
<td>768</td>
<td>1.474</td>
<td>40.95</td>
<td>18.75 \times 10^{-6} at 2810</td>
<td>100</td>
<td>75</td>
<td>40.7</td>
<td>0.855</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>260</td>
<td>2051</td>
<td>693</td>
<td>1.529</td>
<td>37.37</td>
<td>22.69 \times 10^{-6} at 2526</td>
<td>100</td>
<td>72</td>
<td>48.6</td>
<td>0.911</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>237</td>
<td>2884</td>
<td>968</td>
<td>1.619</td>
<td>38.92</td>
<td>21.51 \times 10^{-6} at 2942</td>
<td>60</td>
<td>120</td>
<td>40.5</td>
<td>0.898</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>218</td>
<td>3065</td>
<td>943</td>
<td>1.570</td>
<td>37.70</td>
<td>24.16 \times 10^{-6} at 3273</td>
<td>200</td>
<td>60</td>
<td>44.2</td>
<td>0.781</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>4239</td>
<td>807</td>
<td>1.385</td>
<td>46.79</td>
<td>11.65 \times 10^{-6} at 4370</td>
<td>85</td>
<td>173</td>
<td>27.3</td>
<td>0.848</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Predict the bubble-point pressure by using Standing’s correlation.
Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Coeff. a Equation 2-78</th>
<th>Predicted ( p_b ) Equation 2-77</th>
<th>Measured ( p_b )</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.3613</td>
<td>2181</td>
<td>2392</td>
<td>-8.8</td>
</tr>
<tr>
<td>2</td>
<td>-0.3086</td>
<td>2503</td>
<td>2635</td>
<td>-5.0</td>
</tr>
<tr>
<td>3</td>
<td>-0.3709</td>
<td>1883</td>
<td>2066</td>
<td>-8.8</td>
</tr>
<tr>
<td>4</td>
<td>-0.3115</td>
<td>2896</td>
<td>2899</td>
<td>-0.1</td>
</tr>
<tr>
<td>5</td>
<td>-0.3541</td>
<td>2884</td>
<td>3060</td>
<td>-5.7</td>
</tr>
<tr>
<td>6</td>
<td>-0.1775</td>
<td>3561</td>
<td>4254</td>
<td>-16.3</td>
</tr>
</tbody>
</table>

AAE = 7.4%

McCain (1991) suggested that by replacing the specific gravity of the gas in Equation 2-77 with that of the separator gas, i.e., excluding the gas from the stock tank would improve the accuracy of the equation.

Example 2-24

Using the data of Example 2-23 and given the following separator gas gravities, estimate the bubble-point pressure by applying Standing’s correlation.

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Separator Gas Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.755</td>
</tr>
<tr>
<td>2</td>
<td>0.786</td>
</tr>
<tr>
<td>3</td>
<td>0.801</td>
</tr>
<tr>
<td>4</td>
<td>0.888</td>
</tr>
<tr>
<td>5</td>
<td>0.705</td>
</tr>
<tr>
<td>6</td>
<td>0.813</td>
</tr>
</tbody>
</table>

Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Predicted ( p_b )</th>
<th>Measured ( p_b )</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2411</td>
<td>2392</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>2686</td>
<td>2635</td>
<td>1.93</td>
</tr>
<tr>
<td>3</td>
<td>2098</td>
<td>2066</td>
<td>1.53</td>
</tr>
<tr>
<td>4</td>
<td>2923</td>
<td>2899</td>
<td>0.84</td>
</tr>
<tr>
<td>5</td>
<td>3143</td>
<td>3060</td>
<td>2.70</td>
</tr>
<tr>
<td>6</td>
<td>3689</td>
<td>4254</td>
<td>-13.27</td>
</tr>
</tbody>
</table>

AAE = 3.5%
The Vasquez-Beggs Correlation

Vasquez and Beggs’ gas solubility correlation as presented by Equation 2-71 can be solved for the bubble-point pressure $p_b$ to give:

$$p_b = \left[ \left( C_1 R_s / \gamma_{gs} \right)(10)^a \right]^{C_2}$$

(2-79)

with

$$a = -C_3 \text{ API}/T$$

The gas specific gravity $\gamma_{gs}$ at the reference separator pressure is defined by Equation 2-72. The coefficients $C_1$, $C_2$, and $C_3$ have the following values:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>API ≤ 30</th>
<th>API &gt; 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>27.624</td>
<td>56.18</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.914328</td>
<td>0.84246</td>
</tr>
<tr>
<td>$C_3$</td>
<td>11.172</td>
<td>10.393</td>
</tr>
</tbody>
</table>

Example 2-25

Rework Example 2-23 by applying Equation 2-79.

Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>$\gamma_{gs}$ Equation 2-72</th>
<th>$a$</th>
<th>Predicted $p_b$</th>
<th>Measured $p_b$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.873</td>
<td>-0.689</td>
<td>2319</td>
<td>2392</td>
<td>-3.07</td>
</tr>
<tr>
<td>2</td>
<td>0.855</td>
<td>-0.622</td>
<td>2741</td>
<td>2635</td>
<td>4.03</td>
</tr>
<tr>
<td>3</td>
<td>0.911</td>
<td>-0.702</td>
<td>2043</td>
<td>2066</td>
<td>-1.14</td>
</tr>
<tr>
<td>4</td>
<td>0.850</td>
<td>-0.625</td>
<td>3331</td>
<td>2899</td>
<td>14.91</td>
</tr>
<tr>
<td>5</td>
<td>0.814</td>
<td>-0.678</td>
<td>3049</td>
<td>3060</td>
<td>-0.36</td>
</tr>
<tr>
<td>6</td>
<td>0.834</td>
<td>-0.477</td>
<td>4093</td>
<td>4254</td>
<td>-3.78</td>
</tr>
</tbody>
</table>

AAE= 4.5%
Glaso's Correlation

Glaso (1980) used 45 oil samples, mostly from the North Sea hydrocarbon system, to develop an accurate correlation for bubble-point pressure prediction. Glaso proposed the following expression:

\[
\log(p_b^*) = 1.7669 + 1.7447 \log(p_b^*) - 0.30218 [\log(p_b^*)]^2 \tag{2-80}
\]

where \( p_b^* \) is a correlating number and defined by the following equation:

\[
p_b^* = \left( \frac{R_s}{\gamma_g} \right)^a \left( t \right)^b (API)^c \tag{2-81}
\]

where \( R_s \) = gas solubility, scf/STB
\( t \) = system temperature, °F
\( \gamma_g \) = average specific gravity of the total surface gases
\( a, b, c \) = coefficients of the above equation having the following values:

\[
\begin{align*}
a &= 0.816 \\
b &= 0.172 \\
c &= -0.989
\end{align*}
\]

For volatile oils, Glaso recommends that the temperature exponent \( b \) of Equation 2-81 be slightly changed, to the value of 0.130.

Example 2-26

Resolve Example 2-23 by using Glaso's correlation.

Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>( p_b^* ) Equation 2-81</th>
<th>( p_b^* ) Equation 2-80</th>
<th>Measured ( p_b )</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.51</td>
<td>2431</td>
<td>2392</td>
<td>1.62</td>
</tr>
<tr>
<td>2</td>
<td>16.63</td>
<td>2797</td>
<td>2635</td>
<td>6.14</td>
</tr>
<tr>
<td>3</td>
<td>12.54</td>
<td>2083</td>
<td>2066</td>
<td>0.82</td>
</tr>
<tr>
<td>4</td>
<td>19.30</td>
<td>3240</td>
<td>2899</td>
<td>11.75</td>
</tr>
<tr>
<td>5</td>
<td>19.48</td>
<td>3269</td>
<td>3060</td>
<td>6.83</td>
</tr>
<tr>
<td>6</td>
<td>25.00</td>
<td>4125</td>
<td>4254</td>
<td>-3.04</td>
</tr>
</tbody>
</table>

\[\text{AAE} = 5.03\%\]
Marhoun’s Correlation

Marhoun (1988) used 160 experimentally determined bubble-point pressures from the PVT analysis of 69 Middle Eastern hydrocarbon mixtures to develop a correlation for estimating $p_b$. The author correlated the bubble-point pressure with the gas solubility $R_s$, temperature $T$, and specific gravity of the oil and the gas. Marhoun proposed the following expression:

$$p_b = a R_s^b g^c T^d$$

where

- $T$ = temperature, °R
- $\gamma_o$ = stock-tank oil specific gravity
- $\gamma_g$ = gas specific gravity
- $a$–$e$ = coefficients of the correlation having the following values:
  - $a = 5.38088 \times 10^{-3}$
  - $b = 0.715082$
  - $c = -1.87784$
  - $d = 3.1437$
  - $e = 1.32657$

The reported average absolute relative error for the correlation is 3.66% when compared with the experimental data used to develop the correlation.

Example 2-27

Using Equation 2-82, rework Example 2-23

Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Predicted $p_b$</th>
<th>Measured $p_b$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2417</td>
<td>2392</td>
<td>1.03</td>
</tr>
<tr>
<td>2</td>
<td>2578</td>
<td>2635</td>
<td>-2.16</td>
</tr>
<tr>
<td>3</td>
<td>1992</td>
<td>2066</td>
<td>-3.57</td>
</tr>
<tr>
<td>4</td>
<td>2752</td>
<td>2899</td>
<td>-5.07</td>
</tr>
<tr>
<td>5</td>
<td>3309</td>
<td>3060</td>
<td>8.14</td>
</tr>
<tr>
<td>6</td>
<td>3229</td>
<td>4254</td>
<td>-24.09</td>
</tr>
</tbody>
</table>

AAE = 7.3%
The Petrosky-Farshad Correlation

The Petrosky and Farshad gas solubility equation, i.e., Equation 2-75, can be solved for the bubble-point pressure to give:

\[
p_b = \left[ \frac{112.727 \, R_s^{0.577421}}{T_g^{0.8439} \, (10)^x} \right]^{-1391.051}
\quad (2 - 83)
\]

where the correlating parameter \( x \) is previously defined by Equation 2-75.

The authors concluded that the correlation predicts measured bubble-point pressures with an average absolute error of 3.28%.

Example 2-28

Use the Petrosky and Farshad correlation to predict the bubble-point pressure data given in Example 2-23.

Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>( X )</th>
<th>Predicted ( p_b )</th>
<th>Measured ( p_b )</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2008</td>
<td>2331</td>
<td>2392</td>
<td>-2.55</td>
</tr>
<tr>
<td>2</td>
<td>0.1566</td>
<td>2768</td>
<td>2635</td>
<td>5.04</td>
</tr>
<tr>
<td>3</td>
<td>0.2101</td>
<td>1893</td>
<td>2066</td>
<td>-8.39</td>
</tr>
<tr>
<td>4</td>
<td>0.1579</td>
<td>3156</td>
<td>2899</td>
<td>8.86</td>
</tr>
<tr>
<td>5</td>
<td>0.1900</td>
<td>3288</td>
<td>3060</td>
<td>7.44</td>
</tr>
<tr>
<td>6</td>
<td>0.0667</td>
<td>3908</td>
<td>4254</td>
<td>-8.13</td>
</tr>
</tbody>
</table>

\[ \text{AAE} = 6.74\% \]

Oil Formation Volume Factor

The oil formation volume factor, \( B_o \), is defined as the ratio of the volume of oil (plus the gas in solution) at the prevailing reservoir temperature and pressure to the volume of oil at standard conditions. \( B_o \) is always greater than or equal to unity. The oil formation volume factor can be expressed mathematically as:

\[
B_o = \frac{(V_o)_{p,T}}{(V_o)_{sc}}
\quad (2 - 84)
\]
where \( B_o = \) oil formation volume factor, bbl/STB

\((V_o)_{p,T} = \) volume of oil under reservoir pressure \( p \) and temperature \( T \), bbl

\((V_o)_{sc} = \) volume of oil is measured under standard conditions, STB

A typical oil formation factor curve, as a function of pressure for an undersaturated crude oil \( (p_i > p_b) \), is shown in Figure 2-8. As the pressure is reduced below the initial reservoir pressure \( p_i \), the oil volume increases due to the oil expansion. This behavior results in an increase in the oil formation volume factor and will continue until the bubble-point pressure is reached. At \( p_b \), the oil reaches its maximum expansion and consequently attains a maximum value of \( B_{ob} \) for the oil formation volume factor. As the pressure is reduced below \( p_b \), volume of the oil and \( B_o \) are decreased as the solution gas is liberated. When the pressure is reduced to atmospheric pressure and the temperature to 60°F, the value of \( B_o \) is equal to one.

Most of the published empirical \( B_o \) correlations utilize the following generalized relationship:

\[
B_o = f \left( R_s, \gamma_g, \gamma_o, T \right)
\]

**Figure 2-8.** Oil formation volume factor versus pressure.
Six different methods of predicting the oil formation volume factor are presented below:

- Standing’s correlation
- The Vasquez-Beggs correlation
- Glaso’s correlation
- Marhoun’s correlation
- The Petrosky-Farshad correlation
- Other correlations

It should be noted that all the correlations could be used for any pressure equal to or below the bubble-point pressure.

**Standing’s Correlation**

Standing (1947) presented a graphical correlation for estimating the oil formation volume factor with the gas solubility, gas gravity, oil gravity, and reservoir temperature as the correlating parameters. This graphical correlation originated from examining a total of 105 experimental data points on 22 different California hydrocarbon systems. An average error of 1.2% was reported for the correlation.

Standing (1981) showed that the oil formation volume factor can be expressed more conveniently in a mathematical form by the following equation:

\[
\beta_o = 0.9759 + 0.000120 \left[ R_s \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25(T - 460) \right]^{1.2}
\]  

(2 - 85)

where

- \( T \) = temperature, °R
- \( \gamma_o \) = specific gravity of the stock-tank oil
- \( \gamma_g \) = specific gravity of the solution gas

**The Vasquez-Beggs Correlation**

Vasquez and Beggs (1980) developed a relationship for determining \( B_o \) as a function of \( R_s, \gamma_o, \gamma_g, \) and \( T \). The proposed correlation was based on 6,000 measurements of \( B_o \) at various pressures. Using the regression
analysis technique, Vasquez and Beggs found the following equation to be the best form to reproduce the measured data:

$$B_o = 1.0 + C_1 R_s + (T - 520) \left( \frac{API}{\gamma_{gs}} \right) \left[ C_2 + C_3 R_s \right]$$  \hspace{1cm} (2-86)

where $R = \text{gas solubility, scf/STB}$
$T = \text{temperature, } ^\circ R$
$\gamma_{gs} = \text{gas specific gravity as defined by Equation 2-72}$

Values for the coefficients $C_1$, $C_2$ and $C_3$ are given below:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>API ≤ 30</th>
<th>API &gt; 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$4.677 \times 10^{-4}$</td>
<td>$4.670 \times 10^{-4}$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$1.751 \times 10^{-5}$</td>
<td>$1.100 \times 10^{-5}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$-1.811 \times 10^{-8}$</td>
<td>$1.337 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Vasquez and Beggs reported an average error of 4.7% for the proposed correlation.

**Glaso’s Correlation**

Glaso (1980) proposed the following expressions for calculating the oil formation volume factor:

$$B_o = 1.0 + 10^A$$  \hspace{1cm} (2-87)

where

$$A = -6.58511 + 2.91329 \log B_{ob}^* - 0.27683 (\log B_{ob}^*)^2$$  \hspace{1cm} (2-88)

$B_{ob}^*$ is a *correlating number* and is defined by the following equation:

$$B_{ob}^* = R_s \left( \frac{\gamma_g}{\gamma_o} \right)^{0.526} + 0.968(T - 460)$$  \hspace{1cm} (2 - 89)

where $T = \text{temperature, } ^\circ R$
$\gamma_o = \text{specific gravity of the stock-tank oil}$
The above correlations were originated from studying PVT data on 45 oil samples. The average error of the correlation was reported at \(-0.43\%\) with a standard deviation of \(2.18\%\).

Sutton and Farshad (1984) concluded that Glaso’s correlation offers the best accuracy when compared with the Standing and Vasquez-Beggs correlations. In general, Glaso’s correlation underpredicts formation volume factor. Standing’s expression tends to overpredict oil formation volume factors greater than 1.2 bbl/STB. The Vasquez-Beggs correlation typically overpredicts the oil formation volume factor.

**Marhoun’s Correlation**

Marhoun (1988) developed a correlation for determining the oil formation volume factor as a function of the gas solubility, stock-tank oil gravity, gas gravity, and temperature. The empirical equation was developed by use of the nonlinear multiple regression analysis on 160 experimental data points. The experimental data were obtained from 69 Middle Eastern oil reserves. The author proposed the following expression:

\[
B_o = 0.497069 + 0.862963 \times 10^{-3} T + 0.182594 \times 10^{-2} F + 0.318099 \times 10^{-2} F^2
\]

with the correlating parameter \(F\) as defined by the following equation:

\[
F = R_s^a \gamma_o^b \gamma_g^c
\]

The coefficients \(a\), \(b\) and \(c\) have the following values:

\[
a = 0.742390 \\
b = 0.323294 \\
c = -1.202040
\]

where \(T\) is the system temperature in °R.

**The Petrosky-Farshad Correlation**

Petrosky and Farshad (1993) proposed a new expression for estimating \(B_o\). The proposed relationship is similar to the equation developed by Standing; however, the equation introduces three additional fitting parameters in order to increase the accuracy of the correlation.
The authors used a nonlinear regression model to match experimental crude oil from the Gulf of Mexico hydrocarbon system. Their correlation has the following form:

\[ B_o = 1.0113 + 7.2046 \times 10^{-5} \]

\[
\left[ R_s^{0.3738} \left( \frac{\gamma_g}{\gamma_o^{0.2914}} \right) + 0.24626 (T - 460)^{0.5371} \right]^{3.0936}
\]

where \( T = \) temperature, °R
\( \gamma_o = \) specific gravity of the stock-tank oil

**Material Balance Equation**

Following the definition of \( B_o \) as expressed mathematically by Equation 2-84, it can be shown that:

\[ B_o = \frac{62.4 \gamma_o + 0.0136 R_s \gamma_g}{\rho_o} \]  \( \text{(2 - 93)} \)

where \( \rho_o = \) density of the oil at the specified pressure and temperature, \( \text{lb/ft}^3 \).

The error in calculating \( B_o \) by using Equation 2-93 will depend only on the accuracy of the input variables \( (R_s, \gamma_g, \text{and } \gamma_o) \) and the method of calculating \( \rho_o \).

**Example 2-29**

The following experimental PVT data on six different crude oil systems are available. Results are based on two-stage surface separation.

<table>
<thead>
<tr>
<th>Oil #</th>
<th>T</th>
<th>( P_b )</th>
<th>( R_s )</th>
<th>( B_o )</th>
<th>( \rho_o )</th>
<th>( c_o \text{ at } P &gt; P_b )</th>
<th>( P_{sep} )</th>
<th>( T_{sep} )</th>
<th>API</th>
<th>( \gamma_g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>2377</td>
<td>751</td>
<td>1.528</td>
<td>38.13</td>
<td>( 22.14 \times 10^{-6} \text{ at } 2689 )</td>
<td>150</td>
<td>60</td>
<td>47.1</td>
<td>0.851</td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>2620</td>
<td>768</td>
<td>1.474</td>
<td>40.95</td>
<td>( 18.75 \times 10^{-6} \text{ at } 2810 )</td>
<td>100</td>
<td>75</td>
<td>40.7</td>
<td>0.855</td>
</tr>
<tr>
<td>3</td>
<td>260</td>
<td>2051</td>
<td>693</td>
<td>1.529</td>
<td>37.37</td>
<td>( 22.69 \times 10^{-6} \text{ at } 2526 )</td>
<td>100</td>
<td>72</td>
<td>48.6</td>
<td>0.911</td>
</tr>
<tr>
<td>4</td>
<td>237</td>
<td>2884</td>
<td>968</td>
<td>1.619</td>
<td>38.92</td>
<td>( 21.51 \times 10^{-6} \text{ at } 2942 )</td>
<td>60</td>
<td>120</td>
<td>40.5</td>
<td>0.898</td>
</tr>
<tr>
<td>5</td>
<td>218</td>
<td>3065</td>
<td>943</td>
<td>1.570</td>
<td>37.70</td>
<td>( 24.16 \times 10^{-6} \text{ at } 3273 )</td>
<td>200</td>
<td>60</td>
<td>44.2</td>
<td>0.781</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>4239</td>
<td>807</td>
<td>1.385</td>
<td>46.79</td>
<td>( 11.65 \times 10^{-6} \text{ at } 4370 )</td>
<td>85</td>
<td>173</td>
<td>27.3</td>
<td>0.848</td>
</tr>
</tbody>
</table>
Calculate the oil formation volume factor at the bubble-point pressure by using the six different correlations. Compare the results with the experimental values and calculate the absolute average error (AAE).

**Solution**

<table>
<thead>
<tr>
<th>Crude Oil</th>
<th>Exp. ( B_o )</th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Method 4</th>
<th>Method 5</th>
<th>Method 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.528</td>
<td>1.506</td>
<td>1.474</td>
<td>1.473</td>
<td>1.516</td>
<td>1.552</td>
<td>1.525</td>
</tr>
<tr>
<td>2</td>
<td>1.474</td>
<td>1.487</td>
<td>1.450</td>
<td>1.459</td>
<td>1.477</td>
<td>1.508</td>
<td>1.470</td>
</tr>
<tr>
<td>3</td>
<td>1.529</td>
<td>1.495</td>
<td>1.451</td>
<td>1.461</td>
<td>1.511</td>
<td>1.556</td>
<td>1.542</td>
</tr>
<tr>
<td>4</td>
<td>1.619</td>
<td>1.618</td>
<td>1.542</td>
<td>1.589</td>
<td>1.575</td>
<td>1.632</td>
<td>1.623</td>
</tr>
<tr>
<td>5</td>
<td>1.570</td>
<td>1.571</td>
<td>1.546</td>
<td>1.541</td>
<td>1.554</td>
<td>1.584</td>
<td>1.599</td>
</tr>
<tr>
<td>6</td>
<td>1.385</td>
<td>1.461</td>
<td>1.389</td>
<td>1.438</td>
<td>1.414</td>
<td>1.433</td>
<td>1.387</td>
</tr>
</tbody>
</table>

**%AAE** — 1.7 2.8 2.8 1.3 1.8 0.6

where Method 1 = Standing’s correlation

Method 2 = Vasquez-Beggs correlation

Method 3 = Glaso’s correlation

Method 4 = Marhoun’s correlation

Method 5 = Petrosky-Farshad correlation

Method 6 = Material balance equation

**Isothermal Compressibility Coefficient of Crude Oil**

Isothermal compressibility coefficients are required in solving many reservoir engineering problems, including transient fluid flow problems, and they are also required in the determination of the physical properties of the undersaturated crude oil.

By definition, the isothermal compressibility of a substance is defined mathematically by the following expression:

\[
c = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T
\]

For a crude oil system, the isothermal compressibility coefficient of the oil phase \( c_o \) is defined for pressures above the bubble-point by one of the following equivalent expressions:

\[
c_o = -(1/V)(\partial V/\partial p)_T
\]
\[ c_o = -\left(1/B_o\right)\left(\partial B_o/\partial p\right)_T \quad (2-95) \]
\[ c_o = \left(1/\rho_o\right)\left(\partial \rho_o/\partial p\right)_T \quad (2-96) \]

where \( c_o \) = isothermal compressibility, psi\(^{-1}\)
\( \rho_o \) = oil density lb/ft\(^3\)
\( B_o \) = oil formation volume factor, bbl/STB

At pressures below the bubble-point pressure, the oil compressibility is defined as:

\[ c_o = \frac{-1}{B_o} \left(\partial B_o/\partial p\right) + \frac{B_g}{B_o} \left(\partial R_g/\partial p\right) \quad (2-97) \]

where \( B_g \) = gas formation volume factor, bbl/scf

There are several correlations that are developed to estimate the oil compressibility at pressures above the bubble-point pressure, i.e., undersaturated crude oil system. Three of these correlations are presented below:

• The Vasquez-Beggs correlation
• The Petrosky-Farshad correlation
• McCain’s correlation

The Vasquez-Beggs Correlation

From a total of 4,036 experimental data points used in a linear regression model, Vasquez and Beggs (1980) correlated the isothermal oil compressibility coefficients with \( R_s, T, \, \, ^\circ\text{API}, \, \, \gamma_g, \) and \( p. \) They proposed the following expression:

\[ c_o = \frac{-1.433 + 5R_{sh} + 17.2(T - 460) - 1.180 \gamma_{gs} + 12.61^\circ\text{API}}{10^5 p} \quad (2-98) \]

where
- \( T = \) temperature, \(^\circ\text{R}\)
- \( p = \) pressure above the bubble-point pressure, psia
- \( R_{sh} = \) gas solubility at the bubble-point pressure
- \( \gamma_{gs} = \) corrected gas gravity as defined by Equation 2-72
The Petrosky-Farshad Correlation

Petrosky and Farshad (1993) proposed a relationship for determining the oil compressibility for undersaturated hydrocarbon systems. The equation has the following form:

\[
c_o = 1.705 \times 10^{-7} R_{sb}^{0.69357} T_g^{0.1885} \text{API}^{0.3272}(T - 460)^{0.6729} R_{sb}^{-0.5906}
\]

(2-99)

where

- \( T \) = temperature, °R
- \( R_{sb} \) = gas solubility at the bubble-point pressure, scf/STB

Example 2-30

Using the experimental data given in Example 2-29, estimate the undersaturated oil compressibility coefficient by using the Vasquez-Beggs and the Petrosky-Farshad correlations. Calculate the AAE.

Solution

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Pressure</th>
<th>Measured ( c_o ) ( \times 10^{-4} ) psi</th>
<th>Vasquez-Beggs ( c_o ) ( \times 10^{-4} ) psi</th>
<th>Petrosky-Farshad ( c_o ) ( \times 10^{-4} ) psi</th>
<th>AAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2689</td>
<td>22.14</td>
<td>22.88</td>
<td>22.24</td>
<td>6.18%</td>
</tr>
<tr>
<td>2</td>
<td>2810</td>
<td>18.75</td>
<td>20.16</td>
<td>19.27</td>
<td>4.05%</td>
</tr>
<tr>
<td>3</td>
<td>2326</td>
<td>22.60</td>
<td>23.78</td>
<td>22.92</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2942</td>
<td>21.51</td>
<td>22.31</td>
<td>21.78</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3273</td>
<td>24.16</td>
<td>20.16</td>
<td>20.39</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4370</td>
<td>11.45</td>
<td>11.54</td>
<td>11.77</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.18%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.05%</td>
</tr>
</tbody>
</table>

Below the bubble-point pressure, McCain and coauthors (1988) correlated the oil compressibility with pressure \( p \), the oil API gravity, gas solubility at the bubble-point \( R_{sb} \), and the temperature \( T \) in °R. Their proposed relationship has the following form:

\[
c_o = \exp(A)
\]

(2-100)

where the correlating parameter \( A \) is given by the following expression:

\[
A = -7.633 - 1.497 \ln(p) + 1.115 \ln(T) + 0.533 \ln(\text{API}) + 0.184 \ln(R_{sb})
\]

(2-101)
The authors suggested that the accuracy of the Equation 2-100 can be substantially improved if the bubble-point pressure is known. They improved correlating parameter A by including the bubble-point pressure \( p_b \) as one of the parameters in the above equation, to give:

\[
A = -7.573 - 1.45 \ln (p) - 0.383 \ln (p_b) + 1.402 \ln (T) + 0.256 \ln (\text{API}) + 0.449 \ln (R_{sb}) \tag{2-102}
\]

Analytically, Standing’s correlations for \( R_s \) (Equation 2-70) and \( \beta_o \) (Equation 2-85) can be differentiated with respect to the pressure \( p \) to give:

\[
\frac{\partial R_s}{\partial p} = \frac{R_s}{0.83 p + 21.75} \tag{2 - 103}
\]

\[
\frac{\partial B_o}{\partial p} = \left[ \frac{0.000144 R_s}{(0.83 p + 21.75)} \right] \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} \times \left[ R_s \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25 (T - 460) \right]^{-0.12} \tag{2-104}
\]

The above two expressions can be substituted into Equation 2-97 to give the following relationship:

\[
c_o = \frac{-R_s}{B_o (0.83 p + 21.75)} \times \left\{ 0.00014 \left[ \frac{\gamma_g}{\gamma_o} \right] R_s \left[ \frac{\gamma_g}{\gamma_o} + 1.25 (T - 460) \right]^{-0.12} - B_g \right\} \tag{2-105}
\]

where
- \( p \) = pressure, psia
- \( T \) = temperature, °R
- \( B_g \) = gas formation volume factor at pressure \( p \), bbl/scf
- \( R_s \) = gas solubility at pressure \( p \), scf/STB
- \( B_o \) = oil formation volume factor at \( p \), bbl/STB
- \( \gamma_o \) = specific gravity of the stock-tank oil
- \( \gamma_g \) = specific gravity of the solution gas
Example 2-31

A crude oil system exists at 1650 psi and a temperature of 250°F. The system has the following PVT properties:

- API = 47.1
- \( p_b = 2377 \)
- \( \gamma_g = 0.851 \)
- \( \gamma_{gs} = 0.873 \)
- \( R_{sb} = 751 \text{ scf/STB} \)
- \( B_{ob} = 1.528 \text{ bbl/STB} \)

The laboratory measured oil PVT data at 1650 psig are listed below:

- \( B_o = 1.393 \text{ bbl/STB} \)
- \( R_s = 515 \text{ scf/STB} \)
- \( B_g = 0.001936 \text{ bbl/scf} \)
- \( c_o = 324.8 \times 10^{-6} \text{ psi}^{-1} \)

Estimate the oil compressibility by using:

a. McCain’s correlation
b. Equation 2-105

Solution

McCain’s Correlation:
• Calculate the correlating parameter \( A \) by applying Equation 2-102

\[
A = -7.573 - 1.45 \ln (1665) - 0.383 \ln (2392) + 1.402 \ln (710) \\
+ 0.256 \ln (47.1) + 0.449 \ln (451) = -8.1445
\]

• Solve for \( c_o \) by using Equation 2-100

\[
c_o = \exp (-8.1445) = 290.3 \times 10^{-6} \text{ psi}^{-1}
\]

Oil Compressibility Using Equation 2-105

\[
c_o = \frac{-515}{1.393[0.83(1665) + 21.75]} \\
\times \left\{ 0.00014 \left( \frac{0.851}{0.792} \right) \left( \frac{515}{792} \right)^{0.12} + 1.25(250) - 0.001936 \right\}
\]

\[
c_o = 424 \times 10^{-6} \text{ psi}^{-1}
\]
It should be pointed out that when it is necessary to establish PVT relationships for the hydrocarbon system through correlations or by extrapolation, care should be exercised to see that the PVT functions are consistent.

This consistency is assured if the increase in oil volume with increasing pressure is less than the decrease in volume associated with the gas going into solution. Since the oil compressibility coefficient $c_o$ as expressed by Equation 2-97 must be positive, that leads to the following consistency criteria:

$$\frac{\partial B_o}{\partial p} < B_g \frac{\partial R_g}{\partial p}$$  \hspace{1cm} (2-106)

This consistency can easily be checked in the tabular form of PVT data. The PVT consistency errors most frequently occur at higher pressures where the gas formation volume factor, $B_g$, assumes relatively small values.

**Oil Formation Volume Factor for Undersaturated Oils**

With increasing pressures above the bubble-point pressure, the oil formation volume factor decreases due to the compression of the oil, as illustrated schematically in Figure 2-9.

To account for the effects of oil compression on $B_o$, the oil formation volume factor at the bubble-point pressure is first calculated by using any of the methods previously described. The calculated $B_o$ is then adjusted to account for the effect if increasing the pressure above the bubble-point pressure. This adjustment step is accomplished by using the isothermal compressibility coefficient as described below.

The isothermal compressibility coefficient (as expressed mathematically by Equation 2-94) can be equivalently written in terms of the oil formation volume factor:

$$c_o = \frac{-1}{B_o} \frac{\partial B_o}{\partial p}$$

The above relationship can be rearranged and integrated to produce

$$\int_{p_b}^{p} -c_o \, dp = \int_{B_{ob}}^{B_o} \frac{1}{B_o} \, dB_o$$  \hspace{1cm} (2-107)
Evaluating $c_0$ at the arithmetic average pressure and concluding the integration procedure to give:

$$B_0 = B_{ob} \exp\left[-c_0 (p - p_b)\right] \quad (2-108)$$

where $B_0$ = oil formation volume factor at the pressure of interest,

bbl/STB

$B_{ob}$ = oil formation volume factor at the bubble-point pressure,

bbl/STB

$p = \text{pressure of interest, psia}$

$p_b = \text{bubble-point pressure, psia}$

Replacing with the Vasquez-Beggs’ $c_0$ expression, i.e., Equation 2-98, and integrating the resulting equation gives:

$$B_0 = B_{ob} \exp\left[-A \ln\left(\frac{p}{p_b}\right)\right] \quad (2-109)$$
Replacing \( c_o \) in Equation 2-107 with the Petrosky-Farshad expression (i.e., Equation 2-99) and integrating gives:

\[
B_o = B_{sb} \exp \left[ -A \left( p^{0.4094} - p_b^{0.4094} \right) \right] \tag{2 - 110}
\]

with the correlating parameter \( A \) as defined by:

\[
A = 4.1646 \times 10^{-7} R_{sb}^{0.69357} \gamma_g^{0.1885} (\text{API})^{0.3272} (T - 460)^{0.6729} \tag{2 - 111}
\]

where

- \( T \) = temperature, °R
- \( p \) = pressure, psia
- \( R_{sb} \) = gas solubility at the bubble-point pressure

**Example 2-32**

Using the PVT data given in Example 2-31, calculate the oil formation volume factor at 5000 psig by using:

a. Equation 2-109  
b. Equation 2-110

The experimental measured \( B_o \) is 1.457 bbl/STB.

**Solution**

Using Equation 2-109:

- Calculate the parameter \( A \):

\[
A = 10^{-5} \left[ -1433 + 5 \times 751 + 17.2 \times 250 - 1180 \times 0.873 \right. \\
\left. + 12.61 \times 47.1 \right] = 0.061858
\]

- Apply Equation 2-109:

\[
B_o = 1.528 \exp \left[ -0.061858 \ln \left( \frac{5015}{2392} \right) \right] = 1.459 \text{ bbl/STB}
\]
Using Equation 2-110:
• Calculate the correlating parameter A from Equation 2-111:
\[ A = 4.1646 \times 10^{-7} (751)^{0.69357} (0.851)^{0.1885} (47.1)^{0.3272} \times (250)^{0.6729} = 0.005778 \]

• Solve for \( B_o \) by applying Equation 2-110:
\[ B_o = 1.528 \exp \left[-0.005778 (5015.4094 - 2392.4096)\right] = 1.453 \text{ bbl/STB} \]

**Crude Oil Density**

The crude oil density is defined as the mass of a unit volume of the crude at a specified pressure and temperature. It is usually expressed in pounds per cubic foot. Several empirical correlations for calculating the density of liquids of unknown compositional analysis have been proposed. The correlations employ limited PVT data such as gas gravity, oil gravity, and gas solubility as correlating parameters to estimate liquid density at the prevailing reservoir pressure and temperature.

Equation 2-93 may be used to calculate the density of the oil at pressure below or equal to the bubble-point pressure. Solving Equation 2-93 for the oil density gives:

\[ \rho_o = \frac{62.4 \gamma_o + 0.0136 R_s \gamma_g}{B_o} \]  \hspace{1cm} (2-112)

where
- \( \gamma_o \) = specific gravity of the stock-tank oil
- \( R_s \) = gas solubility, scf/STB
- \( \rho_o \) = oil density, lb/ft³

Standing (1981) proposed an empirical correlation for estimating the oil formation volume factor as a function of the gas solubility \( R_s \), the specific gravity of stock-tank oil \( \gamma_o \), the specific gravity of solution gas \( \gamma_g \), and the system temperature \( T \). By coupling the mathematical definition of the oil formation volume factor (as discussed in a later section) with Standing’s correlation, the density of a crude oil at a specified pressure and temperature can be calculated from the following expression:
where \( T \) = system temperature, °R  
\( g \) = specific gravity of the stock-tank oil

**Example 2-33**

Using the experimental PVT data given in Example 2-29 for the six different crude oil systems, calculate the oil density by using Equations 2-112 and 2-113. Compare the results with the experimental values and calculate the absolute average error (AAE).

**Solution**

<table>
<thead>
<tr>
<th>Crude Oil</th>
<th>Measured Oil Density</th>
<th>Equation 2-112</th>
<th>Equation 2-113</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38.13</td>
<td>38.04</td>
<td>38.31</td>
</tr>
<tr>
<td>2</td>
<td>40.95</td>
<td>40.85</td>
<td>40.18</td>
</tr>
<tr>
<td>3</td>
<td>37.37</td>
<td>37.68</td>
<td>38.26</td>
</tr>
<tr>
<td>4</td>
<td>42.25</td>
<td>41.52</td>
<td>40.39</td>
</tr>
<tr>
<td>5</td>
<td>37.70</td>
<td>38.39</td>
<td>38.08</td>
</tr>
<tr>
<td>6</td>
<td>46.79</td>
<td>46.86</td>
<td>44.11</td>
</tr>
</tbody>
</table>

AAE 0.84% 2.65%

Density of the oil at pressures above the bubble-point pressure can be calculated with:

\[
\rho_o = \rho_{ob} \exp \left[ c_o (p - p_b) \right] 
\]

(2-114)

where \( \rho_o \) = density of the oil at pressure \( p \), lb/ft\(^3\)  
\( \rho_{ob} \) = density of the oil at the bubble-point pressure, lb/ft\(^3\)  
\( c_o \) = isothermal compressibility coefficient at average pressure, psi\(^{-1}\)
Vasquez-Beggs’ oil compressibility correlation and the Petrosky-Farshad $c_o$ expression can be incorporated in Equation 2-114 to give:

**For the Vasquez-Beggs $c_o$ equation:**

$$
\rho_o = \rho_{ob} \exp \left[ A \ln \left( \frac{p}{p_b} \right) \right] 
$$

where

$$
A = 10^{-5} [-1,433 + 5 \ R_{sb} + 17.2 \ (T - 460) - 1,180 \ \gamma_{gs} + 12.61 \ °API] 
$$

**For the Petrosky-Farshad $c_o$ expression:**

$$
\rho_o = \rho_{ob} \exp [A \ (p^{0.4094} - p_b^{0.4094})] 
$$

with the correlating parameter $A$ as given by Equation 2-111

**Crude Oil Viscosity**

Crude oil viscosity is an important physical property that controls and influences the flow of oil through porous media and pipes. The viscosity, in general, is defined as the internal resistance of the fluid to flow.

The oil viscosity is a strong function of the temperature, pressure, oil gravity, gas gravity, and gas solubility. Whenever possible, oil viscosity should be determined by laboratory measurements at reservoir temperature and pressure. The viscosity is usually reported in standard PVT analyses. If such laboratory data are not available, engineers may refer to published correlations, which usually vary in complexity and accuracy depending upon the available data on the crude oil.

According to the pressure, the viscosity of crude oils can be classified into three categories:

- **Dead-Oil Viscosity**
  The dead-oil viscosity is defined as the viscosity of crude oil at atmospheric pressure (no gas in solution) and system temperature.

- **Saturated-Oil Viscosity**
  The saturated (bubble-point)-oil viscosity is defined as the viscosity of the crude oil at the bubble-point pressure and reservoir temperature.
• **Undersaturated-Oil Viscosity**

The undersaturated-oil viscosity is defined as the viscosity of the crude oil at a pressure above the bubble-point and reservoir temperature.

Estimation of the oil viscosity at pressures equal to or below the bubble-point pressure is a two-step procedure:

*Step 1.* Calculate the viscosity of the oil without dissolved gas (dead oil), $\mu_{ob}$, at the reservoir temperature.

*Step 2.* Adjust the dead-oil viscosity to account for the effect of the gas solubility at the pressure of interest.

At pressures greater than the bubble-point pressure of the crude oil, another adjustment step, i.e. Step 3, should be made to the bubble-point oil viscosity, $\mu_{ob}$, to account for the compression and the degree of undersaturation in the reservoir. A brief description of several correlations that are widely used in estimating the oil viscosity in the above three steps is given below.

**METHODS OF CALCULATING VISCOSITY OF THE DEAD OIL**

Several empirical methods are proposed to estimate the viscosity of the dead oil, including:

• Beal’s correlation
• The Beggs-Robinson correlation
• Glaso’s correlation

These three methods are presented below.

**Beal’s Correlation**

From a total of 753 values for dead-oil viscosity at and above 100°F, Beal (1946) developed a graphical correlation for determining the viscosity of the dead oil as a function of temperature and the API gravity of the crude. Standing (1981) expressed the proposed graphical correlation in a mathematical relationship as follows:
\[ \mu_{od} = \left(0.32 + \frac{1.8(10^7)}{API^{0.43} + 8.33} \right) \left(\frac{360}{T - 260}\right)^a \]  

(2 - 117)

with

\[ a = 10^{0.43 + 8.33/\text{API}} \]

where \( \mu_{od} \) = viscosity of the dead oil as measured at 14.7 psia and reservoir temperature, cp

\( T \) = temperature, °R

**The Beggs-Robinson Correlation**

Beggs and Robinson (1975) developed an empirical correlation for determining the viscosity of the dead oil. The correlation originated from analyzing 460 dead-oil viscosity measurements. The proposed relationship is expressed mathematically as follows:

\[ \mu_{od} = 10^x - 1 \]  

(2-118)

where \( x = Y (T - 460)^{-1.163} \)

\[ Y = 10^Z \]

\[ Z = 3.0324 - 0.02023^9 \text{API} \]

An average error of -0.64% with a standard deviation of 13.53% was reported for the correlation when tested against the data used for its development. Sutton and Farshad (1980) reported an error of 114.3% when the correlation was tested against 93 cases from the literature.

**Glaso’s Correlation**

Glaso (1980) proposed a generalized mathematical relationship for computing the dead-oil viscosity. The relationship was developed from experimental measurements on 26 crude oil samples. The correlation has the following form:

\[ \mu_{od} = [3.141 \times 10^{10}] (T - 460)^{-3.444} \log(\text{API})^a \]  

(2 - 119)

where the coefficient \( a \) is given by:

\[ a = 10.313 \log(T - 460) - 36.447 \]
The above expression can be used within the range of 50–300°F for the system temperature and 20–48° for the API gravity of the crude.

Sutton and Farshad (1986) concluded that Glaso’s correlation showed the best accuracy of the three previous correlations.

**METHODS OF CALCULATING THE SATURATED OIL VISCOSITY**

Several empirical methods are proposed to estimate the viscosity of the saturated oil, including:

- The Chew-Connally correlation
- The Beggs-Robinson correlation

These two correlations are presented below.

**The Chew-Connally Correlation**

Chew and Connally (1959) presented a graphical correlation to adjust the dead-oil viscosity according to the gas solubility at saturation pressure. The correlation was developed from 457 crude oil samples. Standing (1977) expressed the correlation in a mathematical form as follows:

\[
\mu_{ob} = (10)^a \mu_{od}^b
\]

(2-120)

with

\[
a = R_s [2.2(10^{-7}) R_s - 7.4(10^{-4})]
\]

\[
b = \frac{0.68}{10^c} + \frac{0.25}{10^d} + \frac{0.062}{10^e}
\]

\[
c = 8.62(10^{-5}) R_s
\]

\[
d = 1.1(10^{-3}) R_s
\]

\[
e = 3.74(10^{-3}) R_s
\]

where \( \mu_{ob} \) = viscosity of the oil at the bubble-point pressure, cp

\( \mu_{od} \) = viscosity of the dead oil at 14.7 psia and reservoir temperature, cp

The experimental data used by Chew and Connally to develop their correlation encompassed the following ranges of values for the independent variables:
Pressure, psia: 132–5,645
Temperature, °F: 72–292
Gas solubility, scf/STB: 51–3,544
Dead oil viscosity, cp: 0.377–50

The Beggs-Robinson Correlation

From 2,073 saturated oil viscosity measurements, Beggs and Robinson (1975) proposed an empirical correlation for estimating the saturated-oil viscosity. The proposed mathematical expression has the following form:

\[ \mu_{ob} = a(\mu_{od})^b \]  

(2-121)

where

\[ a = 10.715(R_s + 100)^{-0.515} \]
\[ b = 5.44(R_s + 150)^{0.338} \]

The reported accuracy of the correlation is –1.83% with a standard deviation of 27.25%.

The ranges of the data used to develop Beggs and Robinson’s equation are:

Pressure, psia: 132–5,265
Temperature, °F: 70–295
API gravity: 16–58
Gas solubility, scf/STB: 20–2,070

METHODS OF CALCULATING THE VISCOSITY OF THE UNDERSATURATED OIL

Oil viscosity at pressures above the bubble point is estimated by first calculating the oil viscosity at its bubble-point pressure and adjusting the bubble-point viscosity to higher pressures. Vasquez and Beggs proposed a simple mathematical expression for estimating the viscosity of the oil above the bubble-point pressure. This method is discussed below.

The Vasquez-Beggs Correlation

From a total of 3,593 data points, Vasquez and Beggs (1980) proposed the following expression for estimating the viscosity of undersaturated crude oil:
\[ \mu_o = \mu_{ob} \left( \frac{p}{p_b} \right)^m \]  

(2-123)

where

\[ m = 2.6 \, p^{1.187} \, 10^a \]

with

\[ a = -3.9(10^{-5}) \, p - 5 \]

The data used in developing the above correlation have the following ranges:

- Pressure, psia: 141–9,151
- Gas solubility, scf/STB: 9.3–2,199
- Viscosity, cp: 0.117–148
- Gas gravity: 0.511–1.351
- API gravity: 15.3–59.5

The average error of the viscosity correlation is reported as −7.54%.

**Example 2-34**

In addition to the experimental PVT data given in Example 2-29, the following viscosity data are available:

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Dead Oil ( \mu_{od} ) at T</th>
<th>Saturated Oil ( \mu_{os} ) at p</th>
<th>Undersaturated Oil ( \mu_u ) at p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.765 @ 250°F</td>
<td>0.224</td>
<td>0.281 @ 5000 psi</td>
</tr>
<tr>
<td>2</td>
<td>1.286 @ 220°F</td>
<td>0.373</td>
<td>0.450 @ 5000 psi</td>
</tr>
<tr>
<td>3</td>
<td>0.686 @ 260°F</td>
<td>0.221</td>
<td>0.292 @ 5000 psi</td>
</tr>
<tr>
<td>4</td>
<td>1.014 @ 237°F</td>
<td>0.377</td>
<td>0.414 @ 6000 psi</td>
</tr>
<tr>
<td>5</td>
<td>1.009 @ 218°F</td>
<td>0.305</td>
<td>0.394 @ 6000 psi</td>
</tr>
<tr>
<td>6</td>
<td>4.166 @ 180°F</td>
<td>0.950</td>
<td>1.008 @ 5000 psi</td>
</tr>
</tbody>
</table>

Using all the oil viscosity correlations discussed in this chapter, please calculate \( \mu_{od} \), \( \mu_{ob} \), and the viscosity of the undersaturated oil.
### Solution

#### Dead-oil viscosity

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Measured $\mu_{ad}$</th>
<th>Beal's</th>
<th>Beggs-Robinson</th>
<th>Glaso's</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.765</td>
<td>0.322</td>
<td>0.568</td>
<td>0.417</td>
</tr>
<tr>
<td>2</td>
<td>0.286</td>
<td>0.638</td>
<td>1.020</td>
<td>0.775</td>
</tr>
<tr>
<td>3</td>
<td>0.686</td>
<td>0.275</td>
<td>0.493</td>
<td>0.363</td>
</tr>
<tr>
<td>4</td>
<td>1.014</td>
<td>0.545</td>
<td>0.917</td>
<td>0.714</td>
</tr>
<tr>
<td>5</td>
<td>1.009</td>
<td>0.512</td>
<td>0.829</td>
<td>0.598</td>
</tr>
<tr>
<td>6</td>
<td>4.166</td>
<td>4.425</td>
<td>4.246</td>
<td>4.536</td>
</tr>
</tbody>
</table>

AAE: 44.9% 17.32% 35.26%

#### Saturated-oil viscosity

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Measured $\mu_{ab}$</th>
<th>Chew-Connally</th>
<th>Beggs-Robinson</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.224</td>
<td>0.313*</td>
<td>0.287*</td>
</tr>
<tr>
<td>2</td>
<td>0.373</td>
<td>0.426</td>
<td>0.377</td>
</tr>
<tr>
<td>3</td>
<td>0.221</td>
<td>0.308</td>
<td>0.279</td>
</tr>
<tr>
<td>4</td>
<td>0.377</td>
<td>0.311</td>
<td>0.297</td>
</tr>
<tr>
<td>5</td>
<td>0.305</td>
<td>0.316</td>
<td>0.300</td>
</tr>
<tr>
<td>6</td>
<td>0.950</td>
<td>0.842</td>
<td>0.689</td>
</tr>
</tbody>
</table>

AAE: 21% 17%

*Using the measured $\mu_{ab}$.  

#### Undersaturated-oil viscosity

<table>
<thead>
<tr>
<th>Oil #</th>
<th>Measured $\mu_p$</th>
<th>Beal's</th>
<th>Vasquez-Beggs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.281</td>
<td>0.273*</td>
<td>0.303*</td>
</tr>
<tr>
<td>2</td>
<td>0.45</td>
<td>0.437</td>
<td>0.485</td>
</tr>
<tr>
<td>3</td>
<td>0.292</td>
<td>0.275</td>
<td>0.318</td>
</tr>
<tr>
<td>4</td>
<td>0.414</td>
<td>0.434</td>
<td>0.472</td>
</tr>
<tr>
<td>5</td>
<td>0.396</td>
<td>0.373</td>
<td>0.417</td>
</tr>
<tr>
<td>6</td>
<td>1.008</td>
<td>0.945</td>
<td>1.016</td>
</tr>
</tbody>
</table>

AAE: 3.8% 7.5%

*Using the measured $\mu_p$. 

---

Reservoir Engineering Handbook
Surface/Interfacial Tension

The surface tension is defined as the force exerted on the boundary layer between a liquid phase and a vapor phase per unit length. This force is caused by differences between the molecular forces in the vapor phase and those in the liquid phase, and also by the imbalance of these forces at the interface. The surface can be measured in the laboratory and is unusually expressed in dynes per centimeter. The surface tension is an important property in reservoir engineering calculations and designing enhanced oil recovery projects.

Sugden (1924) suggested a relationship that correlates the surface tension of a pure liquid in equilibrium with its own vapor. The correlating parameters of the proposed relationship are molecular weight $M$ of the pure component, the densities of both phases, and a newly introduced temperature independent parameter $P_{ch}$. The relationship is expressed mathematically in the following form:

$$\sigma = \left[ \frac{P_{ch} (\rho_L - \rho_v)}{M} \right]^4$$

(2-124)

where $\sigma$ is the surface tension and $P_{ch}$ is a temperature independent parameter and is called the parachor.

The parachor is a dimensionless constant characteristic of a pure compound and is calculated by imposing experimentally measured surface tension and density data on Equation 2-124 and solving for $P_{ch}$. The Parachor values for a selected number of pure compounds are given in Table 2-1 as reported by Weinaug and Katz (1943).

<table>
<thead>
<tr>
<th>Component</th>
<th>Parachor</th>
<th>Component</th>
<th>Parachor</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>78.0</td>
<td>n-C$_4$</td>
<td>189.9</td>
</tr>
<tr>
<td>N$_2$</td>
<td>41.0</td>
<td>i-C$_4$</td>
<td>225.0</td>
</tr>
<tr>
<td>C$_1$</td>
<td>77.0</td>
<td>n-C$_5$</td>
<td>231.5</td>
</tr>
<tr>
<td>C$_2$</td>
<td>108.0</td>
<td>n-C$_6$</td>
<td>271.0</td>
</tr>
<tr>
<td>C$_3$</td>
<td>150.3</td>
<td>n-C$_7$</td>
<td>312.5</td>
</tr>
<tr>
<td>i-C$_4$</td>
<td>181.5</td>
<td>n-C$_8$</td>
<td>351.5</td>
</tr>
</tbody>
</table>
Fanchi (1985) correlated the parachor with molecular weight with a simple linear equation. This linear is only valid for components heavier than methane. Fanchi’s linear equation has the following form:

\[(P_{ch})_i = 69.9 + 2.3 M_i\]  \hspace{1cm} \text{(2-125)}

where \(M_i\) = molecular weight of component \(i\)

\((P_{ch})_i\) = parachor of component \(i\)

For a complex hydrocarbon mixture, Katz et al. (1943) employed the Sugden correlation for mixtures by introducing the compositions of the two phases into Equation 2-124. The modified expression has the following form:

\[\sigma^{1/4} = \sum_{i=1}^{n} \left[(P_{ch})_i (A x_i - B y_i)\right]\]  \hspace{1cm} \text{(2-126)}

with the parameters \(A\) and \(B\) as defined by:

\[A = \frac{\rho_o}{62.4 M_o}\]

\[B = \frac{\rho_g}{62.4 M_g}\]

where \(\rho_o\) = density of the oil phase, lb/ft\(^3\)

\(M_o\) = apparent molecular weight of the oil phase

\(\rho_g\) = density of the gas phase, lb/ft\(^3\)

\(M_g\) = apparent molecular weight of the gas phase

\(x_i\) = mole fraction of component \(i\) in the oil phase

\(y_i\) = mole fraction of component \(i\) in the gas phase

\(n\) = total number of components in the system

**Example 2-35**

The composition of a crude oil and the associated equilibrium gas is given below. The reservoir pressure and temperature are 4,000 psia and 160°F, respectively.
The following additional PVT data are available:

Oil density = 46.23 lb/ft³
Gas density = 18.21 lb/ft³
Molecular weight of C$_{7+}$ = 215

Calculate the surface tension.

**Solution**

*Step 1.* Calculate the apparent molecular weight of the liquid and gas phase:

\[ M_o = 100.253 \quad M_g = 24.99 \]

*Step 2.* Calculate the coefficients A and B:

\[ A = \frac{46.23}{(62.4)(100.253)} = 0.00739 \]

\[ B = \frac{18.21}{(62.6)(24.99)} = 0.01168 \]

*Step 3.* Calculate the parachor of C$_{7+}$ from Equation 2-125:

\[ (P_{cb})_{C_{7+}} = 69.9 + (2.3)(215) = 564.4 \]
Step 4. Construct the following working table:

<table>
<thead>
<tr>
<th>Component</th>
<th>$P_{ch}$</th>
<th>$Ax_1$</th>
<th>$By_1$</th>
<th>$P_{ch}(Ax_1 - By_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>77</td>
<td>0.00333</td>
<td>0.0090</td>
<td>-0.4361</td>
</tr>
<tr>
<td>$C_2$</td>
<td>108</td>
<td>0.00037</td>
<td>0.00093</td>
<td>-0.0605</td>
</tr>
<tr>
<td>$C_3$</td>
<td>150.3</td>
<td>0.00037</td>
<td>0.00070</td>
<td>-0.0497</td>
</tr>
<tr>
<td>n-$C_4$</td>
<td>189.9</td>
<td>0.00022</td>
<td>0.00047</td>
<td>-0.0475</td>
</tr>
<tr>
<td>n-$C_5$</td>
<td>231.5</td>
<td>0.00007</td>
<td>0.00023</td>
<td>-0.0370</td>
</tr>
<tr>
<td>$C_6$</td>
<td>271.0</td>
<td>0.00074</td>
<td>0.00023</td>
<td>-0.0423</td>
</tr>
<tr>
<td>$C_7^+$</td>
<td>564.4</td>
<td>0.00296</td>
<td>0.000117</td>
<td>1.6046</td>
</tr>
</tbody>
</table>

Step 5. $\sigma = (0.9315)^4 = 0.753$ dynes/cm

PROPERTIES OF RESERVOIR WATER

Water Formation Volume Factor

The water formation volume factor can be calculated by the following mathematical expression:*

$$B_w = A_1 + A_2 p + A_3 p^2$$  \hspace{1cm} (2-127)

where the coefficients $A_1 - A_3$ are given by the following expression:

$$A_i = a_1 + a_2(T - 460) + a_3(T - 460)^2$$

with $a_1 - a_3$ given for gas-free and gas-saturated water:

**Gas-Free Water**

<table>
<thead>
<tr>
<th>$A_i$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.9947</td>
<td>5.8(10^{-6})</td>
<td>1.02(10^{-6})</td>
</tr>
<tr>
<td>$A_2$</td>
<td>-4.228(10^{-6})</td>
<td>1.8376(10^{-8})</td>
<td>-6.77(10^{-11})</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1.3(10^{-10})</td>
<td>-1.3855(10^{-12})</td>
<td>4.285(10^{-15})</td>
</tr>
</tbody>
</table>

**Gas-Saturated Water**

<table>
<thead>
<tr>
<th>$A_i$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.9911</td>
<td>6.35(10^{-5})</td>
<td>8.5(10^{-7})</td>
</tr>
<tr>
<td>$A_2$</td>
<td>-1.093(10^{-6})</td>
<td>-3.497(10^{-9})</td>
<td>4.57(10^{-12})</td>
</tr>
<tr>
<td>$A_3$</td>
<td>-5.0(10^{-11})</td>
<td>6.429(10^{-13})</td>
<td>-1.43(10^{-15})</td>
</tr>
</tbody>
</table>

The temperature $T$ in Equation 2-127 is in °R.

**Water Viscosity**

Meehan (1980) proposed a water viscosity correlation that accounts for both the effects of pressure and salinity:

$$\mu_w = \mu_{wd} [1 + 3.5 \times 10^{-2} p^2 (T-40)] \quad (2-128)$$

with

$$\mu_{wd} = A + B/T$$

$$A = 4.518 \times 10^{-2} + 9.313 \times 10^{-7} Y - 3.93 \times 10^{-12} Y^2$$

$$B = 70.634 + 9.576 \times 10^{-10} Y^2$$

where $\mu_w$ = brine viscosity at $p$ and $T$, cp
$\mu_{wd}$ = brine viscosity at $p = 14.7$, T, cp
$p$ = pressure of interest, psia
$T$ = temperature of interest, T °F
$Y$ = water salinity, ppm

Brill and Beggs (1978) presented a simpler equation, which considers only temperature effects:

$$\mu_w = \exp (1.003 - 1.479 \times 10^{-2} T + 1.982 \times 10^{-5} T^2) \quad (2-129)$$

where $T$ is in °F and $\mu_w$ is in cp.

**Gas Solubility in Water**

The following correlation can be used to determine the gas solubility in water:

$$R_{sw} = A + B p + C p^2 \quad (2-130)$$

where

$A = 2.12 + 3.45 (10^{-3}) T - 3.59 (10^{-5}) T^2$

$B = 0.0107 - 5.26 (10^{-5}) T + 1.48 (10^{-7}) T^2$

$C = 8.75 (10^{-7}) + 3.9 (10^{-9}) T - 1.02 (10^{-11}) T^2$

The temperature $T$ in above equations is expressed in °F.
Water Isothermal Compressibility

Brill and Beggs (1978) proposed the following equation for estimating water isothermal compressibility, ignoring the corrections for dissolved gas and solids:

\[ C_w = (C_1 + C_2 T + C_3 T^2) \times 10^{-6} \]  \hspace{1cm} (2-131)

where
\[ C_1 = 3.8546 - 0.000134 p \]
\[ C_2 = -0.01052 + 4.77 \times 10^{-7} p \]
\[ C_3 = 3.9267 \times 10^{-5} - 8.8 \times 10^{-10} p \]
\[ T = ^\circ F \]
\[ p = \text{psia} \]
\[ C_w = \text{psi}^{-1} \]

PROBLEMS

1. Assuming an ideal gas behavior, calculate the density of n-butane at 220°F and 50 psia.
2. Show that:

\[ y_i = \frac{(w_i/M_i)}{\sum_i (w_i/M_i)} \]

3. Given the following gas:

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>0.65</td>
</tr>
<tr>
<td>C_2</td>
<td>0.15</td>
</tr>
<tr>
<td>C_3</td>
<td>0.10</td>
</tr>
<tr>
<td>n-C_4</td>
<td>0.06</td>
</tr>
<tr>
<td>n-C_5</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Calculate:

a. Mole fraction of the gas
b. Apparent molecular weight
c. Specific gravity
d. Specific volume at 300 psia and 120°F by assuming an ideal gas behavior
4. An ideal gas mixture has a density of 1.92 lb/ft³ at 500 psia and 100°F. Calculate the apparent molecular weight of the gas mixture.

5. Using the gas composition as given in Problem 3, and assuming real gas behavior, calculate:
   a. Gas density at 2,000 psia and 150°F
   b. Specific volume at 2,000 psia and 150°F
   c. Gas formation volume factor in scf/ft³

6. A natural gas with a specific gravity of 0.75 has a gas formation volume factor of 0.00529 ft³/scf at the prevailing reservoir pressure and temperature. Calculate the density of the gas.

7. A natural gas has the following composition:

<table>
<thead>
<tr>
<th>Component</th>
<th>yi</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>0.75</td>
</tr>
<tr>
<td>C₂</td>
<td>0.10</td>
</tr>
<tr>
<td>C₃</td>
<td>0.05</td>
</tr>
<tr>
<td>i-C₄</td>
<td>0.04</td>
</tr>
<tr>
<td>n-C₄</td>
<td>0.03</td>
</tr>
<tr>
<td>i-C₅</td>
<td>0.02</td>
</tr>
<tr>
<td>n-C₅</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Reservoir conditions are 3,500 psia and 200°F. Calculate:
   a. Isothermal gas compressibility coefficient
   b. Gas viscosity by using the
      1. Carr-Kobayashi-Burrows method
      2. Lee-Gonzales-Eakin method

8. Given the following gas composition:

<table>
<thead>
<tr>
<th>Component</th>
<th>yi</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>0.06</td>
</tr>
<tr>
<td>N₂</td>
<td>0.03</td>
</tr>
<tr>
<td>C₁</td>
<td>0.75</td>
</tr>
<tr>
<td>C₂</td>
<td>0.07</td>
</tr>
<tr>
<td>C₃</td>
<td>0.04</td>
</tr>
<tr>
<td>n-C₄</td>
<td>0.03</td>
</tr>
<tr>
<td>n-C₅</td>
<td>0.02</td>
</tr>
</tbody>
</table>

If the reservoir pressure and temperature are 2,500 psia and 175°F, respectively, calculate:
a. Gas density by accounting for the presence of nonhydrocarbon components by using the
   1. Wichert-Aziz method
   2. Carr-Kobayashi-Burrows method
b. Isothermal gas compressibility coefficient
c. Gas viscosity by using the
   1. Carr-Kobayashi-Burrows method
   2. Lee-Gonzales-Eakin method

9. A crude oil system exists at its bubble-point pressure of 1,708.7 psia and a temperature of 131°F. Given the following data:
   API = 40°
   Average specific gravity of separator gas = 0.85
   Separator pressure = 100 psig
   a. Calculate \( R_{sb} \) by using
      1. Standing’s correlation
      2. The Vasquez-Beggs method
      3. Glaso’s correlation
      4. Marhoun’s equation
      5. The Petrosky-Farshad correlation
   b. Calculate \( B_{ob} \) by applying methods listed in Part a.

10. Estimate the bubble-point pressure of a crude oil system with the following limited PVT data:
    API = 35°
    \( T = 160°F \)
    \( R_{sb} = 700 \text{ scf/STB} \)
    \( \gamma_g = 0.75 \)
    Use the six different methods listed in Problem 9.

11. A crude oil system exists at an initial reservoir pressure of 4500 psi and 85°F. The bubble-point pressure is estimated at 2109 psi. The oil properties at the bubble-point pressure are as follows:
    \( B_{ob} = 1.406 \text{ bbl/STB} \)
    \( R_{sb} = 692 \text{ scf/STB} \)
    \( \gamma_g = 0.876 \)
    \( \text{API} = 41.9° \)
    Calculate:
    a. Oil density at the bubble-point pressure
    b. Oil density at 4,500 psi
    c. \( B_o \) at 4500 psi

12. A high-pressure cell has a volume of 0.33 ft\(^3\) and contains gas at 2,500 psia and 130°F, at which conditions its z-factor is 0.75. When 43.6 scf of the gas are bled from the cell, the pressure dropped to
1,000 psia, the temperature remaining at 130°F. What is the gas deviation factor at 1,000 psia and 130°F?
13. A hydrocarbon gas mixture with a specific gravity of 0.7 has a density of 9 lb/ft³ at the prevailing reservoir pressure and temperature. Calculate the gas formation volume factor in bbl/scf.
14. A gas reservoir exists at a 150°F. The gas has the following composition:

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole%</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>89</td>
</tr>
<tr>
<td>C₂</td>
<td>7</td>
</tr>
<tr>
<td>C₃</td>
<td>4</td>
</tr>
</tbody>
</table>

The gas expansion factor $E_g$ was calculated as 204.648 scf/ft³ at the existing reservoir pressure and temperature. Calculate the viscosity of the gas.
15. A 20 cu ft tank at a pressure of 2500 psia and 212°F contains ethane gas. How many pounds of ethane are in the tank?
16. The PVT data as shown below were obtained on a crude oil sample taken from the Nameless Field. The initial reservoir pressure was 3600 psia at 160°F. The average specific gravity of the solution gas is 0.65. The reservoir contains 250 mm bbl of oil initially in place. The oil has a bubble-point pressure of 2500 psi.
   a. Calculate the two-phase oil formation volume factor at:
      1. 3200 psia
      2. 2800 psia
      3. 1800 psia
   b. What is the initial volume of dissolved gas in the reservoir?
   c. Oil compressibility coefficient at 3200 psia.

<table>
<thead>
<tr>
<th>Pressure, psia</th>
<th>Solution gas, scf/STB at 1407 psia and 60°F</th>
<th>Formation Volume Factor, bbl/STB</th>
</tr>
</thead>
<tbody>
<tr>
<td>3600</td>
<td></td>
<td>1.310</td>
</tr>
<tr>
<td>3200</td>
<td></td>
<td>1.317</td>
</tr>
<tr>
<td>2800</td>
<td></td>
<td>1.325</td>
</tr>
<tr>
<td>2500</td>
<td>567</td>
<td>1.333</td>
</tr>
<tr>
<td>2400</td>
<td>554</td>
<td>1.310</td>
</tr>
<tr>
<td>1800</td>
<td>436</td>
<td>1.263</td>
</tr>
<tr>
<td>1200</td>
<td>337</td>
<td>1.210</td>
</tr>
<tr>
<td>600</td>
<td>223</td>
<td>1.140</td>
</tr>
<tr>
<td>200</td>
<td>143</td>
<td>1.070</td>
</tr>
</tbody>
</table>
17. The following PVT data were obtained from the analysis of a bottom-hole sample.

<table>
<thead>
<tr>
<th>$p_{\text{psia}}$</th>
<th>Relative Volume $V/V_{\text{sat}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2927</td>
<td>1.0063</td>
</tr>
<tr>
<td>2703</td>
<td>1.0286</td>
</tr>
<tr>
<td>2199</td>
<td>1.1043</td>
</tr>
<tr>
<td>1610</td>
<td>1.2786</td>
</tr>
<tr>
<td>1206</td>
<td>1.5243</td>
</tr>
<tr>
<td>999</td>
<td>1.7399</td>
</tr>
</tbody>
</table>

a. Plot the $Y$-function versus pressure on rectangular coordinate paper, see Equation 3-3
b. Determine the constants in the equation

$$Y = mp + b$$

c. Recalculate relative oil volume from the equation, see Equation 3-5

18. A 295-cc crude oil sample was placed in a PVT at an initial pressure of 3500 psi. The cell temperature was held at a constant temperature of 220°F. A differential liberation test was then performed on the crude oil sample with the recorded measurements as given below:

<table>
<thead>
<tr>
<th>$p$, psi</th>
<th>$T$, °F</th>
<th>Total Volume, cc</th>
<th>Vol. of Liquids, cc</th>
<th>Vol. of Liberated Gas, scf</th>
<th>Specific Gravity of Liberated Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>3500</td>
<td>220</td>
<td>290</td>
<td>290</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>3300</td>
<td>220</td>
<td>294</td>
<td>294</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>*3000</td>
<td>220</td>
<td>300</td>
<td>300</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>2000</td>
<td>220</td>
<td>323.2</td>
<td>286.4</td>
<td>0.1627</td>
<td>0.823</td>
</tr>
<tr>
<td>1000</td>
<td>220</td>
<td>375.2</td>
<td>271.5</td>
<td>0.1840</td>
<td>0.823</td>
</tr>
<tr>
<td>14.7</td>
<td>60</td>
<td>—</td>
<td>179.53</td>
<td>0.5488</td>
<td>0.823</td>
</tr>
</tbody>
</table>

*Bubble-point pressure

Using the bore-recorded measurements and assuming an oil gravity of 40° API, calculate the following PVT properties:

a. Oil formation volume factor at 3500 psi.
b. Gas solubility at 3500 psi.
c. Oil viscosity at 3500 psi.
d. Isothermal compressibility coefficient at 3300 psi.
e. Oil density at 1000 psi.

19. Experiments were made on a bottom-hole crude oil sample taken from the North Grieve Field to determine the gas solubility and oil formation volume factor as a function of pressure. The initial reservoir pressure was recorded as 3600 psia and reservoir temperature was 130°F. The following data were obtained from the measurements:

<table>
<thead>
<tr>
<th>Pressure (psia)</th>
<th>R_S (scf/STB)</th>
<th>B_o (bbl/STB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3600</td>
<td>567</td>
<td>1.310</td>
</tr>
<tr>
<td>3200</td>
<td>567</td>
<td>1.317</td>
</tr>
<tr>
<td>2800</td>
<td>567</td>
<td>1.325</td>
</tr>
<tr>
<td>2500</td>
<td>567</td>
<td>1.333</td>
</tr>
<tr>
<td>2400</td>
<td>554</td>
<td>1.310</td>
</tr>
<tr>
<td>1800</td>
<td>436</td>
<td>1.263</td>
</tr>
<tr>
<td>1200</td>
<td>337</td>
<td>1.210</td>
</tr>
<tr>
<td>600</td>
<td>223</td>
<td>1.140</td>
</tr>
<tr>
<td>200</td>
<td>143</td>
<td>1.070</td>
</tr>
</tbody>
</table>

At the end of the experiments, the API gravity of the oil was measured as 40°. If the average specific gravity of the solution gas is 0.7, calculate:

a. Total formation volume factor at 3200 psia
b. Oil viscosity at 3200 psia
c. Isothermal compressibility coefficient at 1800 psia

20. You are producing a 35°API crude oil from a reservoir at 5000 psia and 140°F. The bubble-point pressure of the reservoir liquids is 4000 psia at 140°F. Gas with a gravity of 0.7 is produced with the oil at a rate of 900 scf/STB. Calculate:

a. Density of the oil at 5000 psia and 140°F
b. Total formation volume factor at 5000 psia and 140°F

21. An undersaturated-oil reservoir exists at an initial reservoir pressure 3112 psia and a reservoir temperature of 125°F. The bubble point of the oil is 1725 psia. The crude oil has the following pressure versus oil formation volume factor relationship:
The API gravity of the crude oil and the specific gravity of the solution gas are 40° and 0.65, respectively. Calculate the density of the crude oil at 3112 psia and 125°F.

22. A PVT cell contains 320 cc of oil and its bubble-point pressure of 2500 psia and 200°F. When the pressure was reduced to 2000 psia, the volume increased to 335.2 cc. The gas was bled off and found to occupy a volume of 0.145 scf. The volume of the oil was recorded as 303 cc. The pressure was reduced to 14.7 psia and the temperature to 60°F while 0.58 scf of gas was evolved leaving 230 cc of oil with a gravity of 42°API. Calculate:

a. Gas compressibility factor at 2000 psia  
b. Gas solubility at 2000 psia

REFERENCES


The material of which a petroleum reservoir rock may be composed can range from very loose and unconsolidated sand to a very hard and dense sandstone, limestone, or dolomite. The grains may be bonded together with a number of materials, the most common of which are silica, calcite, or clay. Knowledge of the physical properties of the rock and the existing interaction between the hydrocarbon system and the formation is essential in understanding and evaluating the performance of a given reservoir.

Rock properties are determined by performing laboratory analyses on cores from the reservoir to be evaluated. The cores are removed from the reservoir environment, with subsequent changes in the core bulk volume, pore volume, reservoir fluid saturations, and, sometimes, formation wettability. The effect of these changes on rock properties may range from negligible to substantial, depending on characteristics of the formation and property of interest, and should be evaluated in the testing program.

There are basically two main categories of core analysis tests that are performed on core samples regarding physical properties of reservoir rocks. These are:

**Routine core analysis tests**

- Porosity
- Permeability
- Saturation
Special tests

• Overburden pressure
• Capillary pressure
• Relative permeability
• Wettability
• Surface and interfacial tension

The above rock property data are essential for reservoir engineering calculations as they directly affect both the quantity and the distribution of hydrocarbons and, when combined with fluid properties, control the flow of the existing phases (i.e., gas, oil, and water) within the reservoir.

### POROSITY

The porosity of a rock is a measure of the storage capacity (pore volume) that is capable of holding fluids. Quantitatively, the porosity is the ratio of the pore volume to the total volume (bulk volume). This important rock property is determined mathematically by the following generalized relationship:

\[
\phi = \frac{\text{pore volume}}{\text{bulk volume}}
\]

where \( \phi \) = porosity

As the sediments were deposited and the rocks were being formed during past geological times, some void spaces that developed became isolated from the other void spaces by excessive cementation. Thus, many of the void spaces are interconnected while some of the pore spaces are completely isolated. This leads to two distinct types of porosity, namely:

• Absolute porosity
• Effective porosity

### Absolute porosity

The absolute porosity is defined as the ratio of the total pore space in the rock to that of the bulk volume. A rock may have considerable absolute porosity and yet have no conductivity to fluid for lack of pore
interconnection. The absolute porosity is generally expressed mathematically by the following relationships:

\[
\phi_a = \frac{\text{total pore volume}}{\text{bulk volume}}
\]

or

\[
\phi_a = \frac{\text{bulk volume} - \text{grain volume}}{\text{bulk volume}}
\]

where \( \phi_a \) = absolute porosity.

**Effective porosity**

The effective porosity is the percentage of interconnected pore space with respect to the bulk volume, or

\[
\phi = \frac{\text{interconnected pore volume}}{\text{bulk volume}}
\]

where \( \phi \) = effective porosity.

The effective porosity is the value that is used in all reservoir engineering calculations because it represents the interconnected pore space that contains the recoverable hydrocarbon fluids.

Porosity may be classified according to the mode of origin as original induced.

The *original* porosity is that developed in the deposition of the material, while *induced* porosity is that developed by some geologic process subsequent to deposition of the rock. The intergranular porosity of sandstones and the intercrystalline and oolitic porosity of some limestones typify original porosity. Induced porosity is typified by fracture development as found in shales and limestones and by the slugs or solution cavities commonly found in limestones. Rocks having original porosity are more uniform in their characteristics than those rocks in which a large part of the porosity is included. For direct quantitative measurement of porosity, reliance must be placed on formation samples obtained by coring.

Since effective porosity is the porosity value of interest to the petroleum engineer, particular attention should be paid to the methods used to
determine porosity. For example, if the porosity of a rock sample was
determined by saturating the rock sample 100 percent with a fluid of
known density and then determining, by weighing, the increased weight
due to the saturating fluid, this would yield an effective porosity measure-
ment because the saturating fluid could enter only the interconnected pore
spaces. On the other hand, if the rock sample were crushed with a mortar
and pestle to determine the actual volume of the solids in the core sample,
then an absolute porosity measurement would result because the identity
of any isolated pores would be lost in the crushing process.

One important application of the effective porosity is its use in deter-
mining the original hydrocarbon volume in place. Consider a reservoir
with an areal extent of A acres and an average thickness of h feet. The
total bulk volume of the reservoir can be determined from the following
expressions:

\[
\text{Bulk volume} = 43,560 Ah, \quad \text{ft}^3 \tag{4-4}
\]

or

\[
\text{Bulk volume} = 7,758 Ah, \quad \text{bbl} \tag{4-5}
\]

where \( A = \) areal extent, acres
\( h = \) average thickness

The reservoir pore volume PV can then be determined by combining
Equations 4-4 and 4-5 with 4-3. Expressing the reservoir pore volume in
cubic feet gives:

\[
PV = 43,560 Ah\phi, \quad \text{ft}^3 \tag{4-6}
\]

Expressing the reservoir pore volume in barrels gives:

\[
PV = 7,758 Ah\phi, \quad \text{bbl} \tag{4-7}
\]

**Example 4-1**

An oil reservoir exists at its bubble-point pressure of 3000 psia and
temperature of 160°F. The oil has an API gravity of 42° and gas-oil ratio
of 600 scf/STB. The specific gravity of the solution gas is 0.65. The fol-
lowing additional data are also available:
• Reservoir area = 640 acres
• Average thickness = 10 ft
• Connate water saturation = 0.25
• Effective porosity = 15%

Calculate the initial oil in place in STB.

Solution

Step 1. Determine the specific gravity of the stock-tank oil from Equation 2-68.

\[ \gamma_o = \frac{141.5}{42 + 131.5} = 0.8156 \]

Step 2. Calculate the initial oil formation volume factor by applying Standing’s equation, i.e., Equation 2-85, to give:

\[ \beta_o = 0.9759 + 0.00012 \left[ 600 \left( \frac{0.65}{0.8156} \right)^{0.5} + 1.25 \times 160 \right]^{1.2} \]

\[ = 1.306 \text{ bbl/STB} \]

Step 3. Calculate the pore volume from Equation 4-7.

Pore volume = 7758 (640) (10) (0.15) = 7,447,680 bbl

Step 4. Calculate the initial oil in place.

Initial oil in place = 12,412,800 (1 – 0.25)/1.306 = 4,276,998 STB

The reservoir rock may generally show large variations in porosity vertically but does not show very great variations in porosity parallel to the bedding planes. In this case, the arithmetic average porosity or the thickness-weighted average porosity is used to describe the average reservoir porosity. A change in sedimentation or depositional conditions, however, can cause the porosity in one portion of the reservoir to be greatly different from that in another area. In such cases, the areal-weighted average or the volume-weighted average porosity is used to characterize the average rock porosity. These averaging techniques are expressed mathematically in the following forms:
Arithmetic average  \( \phi = \Sigma \phi_i / n \)  \( (4-8) \)
Thickness-weighted average  \( \phi = \Sigma \phi_i h_i / \Sigma h_i \)  \( (4-9) \)
Areal-weighted average  \( \phi = \Sigma \phi_i A_i / \Sigma A_i \)  \( (4-10) \)
Volumetric-weighted average  \( \phi = \Sigma \phi_i A_i h_i / \Sigma A_i h_i \)  \( (4-11) \)

where  
- \( n \) = total number of core samples
- \( h_i \) = thickness of core sample \( i \) or reservoir area \( i \)
- \( \phi_i \) = porosity of core sample \( i \) or reservoir area \( i \)
- \( A_i \) = reservoir area \( i \)

Example 4-2

Calculate the arithmetic average and thickness-weighted average from the following measurements:

<table>
<thead>
<tr>
<th>Sample</th>
<th>Thickness, ft</th>
<th>Porosity, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>2.1</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>1.1</td>
<td>10</td>
</tr>
</tbody>
</table>

Solution

• Arithmetic average

\[
\phi = \frac{10 + 12 + 11 + 13 + 14 + 10}{6} = 11.67\% 
\]

• Thickness-weighted average

\[
\phi = \frac{(1)(10) + (1.5)(12) + (1)(11) + (2)(13) + (2.1)(14) + (1.1)(10)}{1 + 1.5 + 1 + 2 + 2.1 + 1.1} \\
= 12.11\%
\]
SATURATION

Saturation is defined as that fraction, or percent, of the pore volume occupied by a particular fluid (oil, gas, or water). This property is expressed mathematically by the following relationship:

\[
\text{fluid saturation} = \frac{\text{total volume of the fluid}}{\text{pore volume}}
\]

Applying the above mathematical concept of saturation to each reservoir fluid gives

\[
S_o = \frac{\text{volume of oil}}{\text{pore volume}} \quad (4-12)
\]

\[
S_g = \frac{\text{volume of gas}}{\text{pore volume}} \quad (4-13)
\]

\[
S_w = \frac{\text{volume of water}}{\text{pore volume}} \quad (4-14)
\]

where \( S_o \) = oil saturation
\( S_g \) = gas saturation
\( S_w \) = water saturation

Thus, all saturation values are based on pore volume and not on the gross reservoir volume.

The saturation of each individual phase ranges between zero to 100 percent. By definition, the sum of the saturations is 100%, therefore

\[
S_g + S_o + S_w = 1.0 \quad (4-15)
\]

The fluids in most reservoirs are believed to have reached a state of equilibrium and, therefore, will have become separated according to their
density, i.e., oil overlain by gas and underlain by water. In addition to the bottom (or edge) water, there will be connate water distributed throughout the oil and gas zones. The water in these zones will have been reduced to some irreducible minimum. The forces retaining the water in the oil and gas zones are referred to as capillary forces because they are important only in pore spaces of capillary size.

Connate (interstitial) water saturation $S_{wc}$ is important primarily because it reduces the amount of space available between oil and gas. It is generally not uniformly distributed throughout the reservoir but varies with permeability, lithology, and height above the free water table.

Another particular phase saturation of interest is called the critical saturation and it is associated with each reservoir fluid. The definition and the significance of the critical saturation for each phase is described below.

**Critical oil saturation, $S_{oc}$**

For the oil phase to flow, the saturation of the oil must exceed a certain value which is termed critical oil saturation. At this particular saturation, the oil remains in the pores and, for all practical purposes, will not flow.

**Residual oil saturation, $S_{or}$**

During the displacing process of the crude oil system from the porous media by water or gas injection (or encroachment) there will be some remaining oil left that is quantitatively characterized by a saturation value that is larger than the critical oil saturation. This saturation value is called the residual oil saturation, $S_{or}$. The term residual saturation is usually associated with the nonwetting phase when it is being displaced by a wetting phase.

**Movable oil saturation, $S_{om}$**

Movable oil saturation $S_{om}$ is another saturation of interest and is defined as the fraction of pore volume occupied by movable oil as expressed by the following equation:

$$S_{om} = 1 - S_{wc} - S_{oc}$$

where $S_{wc} = $ connate water saturation

$S_{oc} = $ critical oil saturation
Critical gas saturation, $S_{gc}$

As the reservoir pressure declines below the bubble-point pressure, gas evolves from the oil phase and consequently the saturation of the gas increases as the reservoir pressure declines. The gas phase remains immobile until its saturation exceeds a certain saturation, called critical gas saturation, above which gas begins to move.

Critical water saturation, $S_{wc}$

The critical water saturation, connate water saturation, and irreducible water saturation are extensively used interchangeably to define the maximum water saturation at which the water phase will remain immobile.

**Average Saturation**

Proper averaging of saturation data requires that the saturation values be weighted by both the interval thickness $h_i$ and interval porosity $\phi_i$. The average saturation of each reservoir fluid is calculated from the following equations:

$$S_o = \frac{\sum_{i=1}^{n} \phi_i h_i S_{oi}}{\sum_{i=1}^{n} \phi_i h_i} \quad (4-16)$$

$$S_w = \frac{\sum_{i=1}^{n} \phi_i h_i S_{wi}}{\sum_{i=1}^{n} \phi_i h_i} \quad (4-17)$$

$$S_g = \frac{\sum_{i=1}^{n} \phi_i h_i S_{gi}}{\sum_{i=1}^{n} \phi_i h_i} \quad (4-18)$$
where the subscript \( i \) refers to any individual measurement and \( h_i \) represents the depth interval to which \( \phi_i \), \( S_{oi} \), \( S_{gi} \), and \( S_{wi} \) apply.

**Example 4-3**

Calculate average oil and connate water saturation from the following measurements:

<table>
<thead>
<tr>
<th>Sample</th>
<th>( h_i ), ft</th>
<th>( \phi_i ), %</th>
<th>( S_{oi} ), %</th>
<th>( S_{wc} ), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>10</td>
<td>75</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>12</td>
<td>77</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>11</td>
<td>79</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>13</td>
<td>74</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>2.1</td>
<td>14</td>
<td>78</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>1.1</td>
<td>10</td>
<td>75</td>
<td>25</td>
</tr>
</tbody>
</table>

**Solution**

Construct the following table and calculate the average saturation for the oil and water phase:

<table>
<thead>
<tr>
<th>Sample</th>
<th>( h_i ), ft</th>
<th>( \phi ), %</th>
<th>( \phi h )</th>
<th>( S_o )</th>
<th>( S_{o \phi h} )</th>
<th>( S_{wc} )</th>
<th>( S_{wc \phi h} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>.10</td>
<td>.100</td>
<td>.75</td>
<td>.0750</td>
<td>.25</td>
<td>.0250</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>.12</td>
<td>.180</td>
<td>.77</td>
<td>.1386</td>
<td>.23</td>
<td>.0414</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>.11</td>
<td>.110</td>
<td>.79</td>
<td>.0869</td>
<td>.21</td>
<td>.0231</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>.13</td>
<td>.260</td>
<td>.74</td>
<td>.1924</td>
<td>.26</td>
<td>.0676</td>
</tr>
<tr>
<td>5</td>
<td>2.1</td>
<td>.14</td>
<td>.294</td>
<td>.78</td>
<td>.2293</td>
<td>.22</td>
<td>.0647</td>
</tr>
<tr>
<td>6</td>
<td>1.1</td>
<td>.10</td>
<td>.110</td>
<td>.75</td>
<td>.0825</td>
<td>.25</td>
<td>.0275</td>
</tr>
</tbody>
</table>

\[ \text{Oil Saturation: } S_o = \frac{0.8047}{1.054} = 0.7635 \]

\[ \text{Water Saturation: } S_w = \frac{0.2493}{1.054} = 0.2365 \]
WETTABILITY

Wettability is defined as the tendency of one fluid to spread on or adhere to a solid surface in the presence of other immiscible fluids. The concept of wettability is illustrated in Figure 4-1. Small drops of three liquids—mercury, oil, and water—are placed on a clean glass plate. The three droplets are then observed from one side as illustrated in Figure 4-1. It is noted that the mercury retains a spherical shape, the oil droplet develops an approximately hemispherical shape, but the water tends to spread over the glass surface.

The tendency of a liquid to spread over the surface of a solid is an indication of the wetting characteristics of the liquid for the solid. This spreading tendency can be expressed more conveniently by measuring the angle of contact at the liquid-solid surface. This angle, which is always measured through the liquid to the solid, is called the contact angle \( \theta \).

The contact angle \( \theta \) has achieved significance as a measure of wettability. As shown in Figure 4-1, as the contact angle decreases, the wetting characteristics of the liquid increase. Complete wettability would be evidenced by a zero contact angle, and complete nonwetting would be evidenced by a contact angle of 180°. There have been various definitions of intermediate wettability but, in much of the published literature, contact angles of 60° to 90° will tend to repel the liquid.

The wettability of reservoir rocks to the fluids is important in that the distribution of the fluids in the porous media is a function of wettability. Because of the attractive forces, the wetting phase tends to occupy the smaller pores of the rock and the nonwetting phase occupies the more open channels.

![Figure 4-1. Illustration of wettability.](image-url)
SURFACE AND INTERFACIAL TENSION

In dealing with multiphase systems, it is necessary to consider the effect of the forces at the interface when two immiscible fluids are in contact. When these two fluids are liquid and gas, the term surface tension is used to describe the forces acting on the interface. When the interface is between two liquids, the acting forces are called interfacial tension.

Surfaces of liquids are usually blanketed with what acts as a thin film. Although this apparent film possesses little strength, it nevertheless acts like a thin membrane and resists being broken. This is believed to be caused by attraction between molecules within a given system. All molecules are attracted one to the other in proportion to the product of their masses and inversely as the squares of the distance between them.

Consider the two immiscible fluids, air (or gas) and water (or oil) as shown schematically in Figure 4-2. A liquid molecule, which is remote from the interface, is surrounded by other liquid molecules, thus having a resulting net attractive force on the molecule of zero. A molecule at the interface, however, has a force acting on it from the air (gas) molecules lying immediately above the interface and from liquid molecules lying below the interface.

Resulting forces are unbalanced and give rise to surface tension. The unbalanced attraction force between the molecules creates a membrane-like surface with a measurable tension, i.e., surface tension. As a matter

---

**Figure 4-2.** Illustration of surface tension. (After Clark, N. J., Elements of Petroleum Reservoirs, SPE, 1969.)
of fact, if carefully placed, a needle will float on the surface of the liquid, supported by the thin membrane even though it is considerably more dense than the liquid.

The surface or interfacial tension has the units of force per unit of length, e.g., dynes/cm, and is usually denoted by the symbol $\sigma$.

If a glass capillary tube is placed in a large open vessel containing water, the combination of surface tension and wettability of tube to water will cause water to rise in the tube above the water level in the container outside the tube as shown in Figure 4-3.

The water will rise in the tube until the total force acting to pull the liquid upward is balanced by the weight of the column of liquid being

![Figure 4-3. Pressure relations in capillary tubes.](image)
supported in the tube. Assuming the radius of the capillary tube is \( r \), the total upward force \( F_{up} \), which holds the liquid up, is equal to the force per unit length of surface times the total length of surface, or

\[
F_{up} = (2\pi r) \left(\sigma_{gw}\right) (\cos \theta)
\]  

where \( \sigma_{gw} \) = surface tension between air (gas) and water (oil), dyne/cm

\( \theta \) = contact angle

\( r \) = radius, cm

The upward force is counteracted by the weight of the water, which is equivalent to a downward force of mass times acceleration, or

\[
F_{down} = \pi r^2 h \left(\rho_w - \rho_{air}\right) g
\]  

where \( h \) = height to which the liquid is held, cm

\( g \) = acceleration due to gravity, cm/sec\(^2\)

\( \rho_w \) = density of water, gm/cm\(^3\)

\( \rho_{air} \) = density of gas, gm/cm\(^3\)

Because the density of air is negligible in comparison with the density of water, Equation 4-20 is reduced to:

\[
F_{down} = \pi r^2 \rho_w g
\]  

Equating Equation 4-19 with 4-21 and solving for the surface tension gives:

\[
\sigma_{gw} = \frac{r h \rho_w g}{2 \cos \theta}
\]  

The generality of Equations 4-19 through 4-22 will not be lost by applying them to the behavior of two liquids, i.e., water and oil. Because the density of oil is not negligible, Equation 4-22 becomes

\[
\sigma_{ow} = \frac{r h g \left(\rho_w - \rho_o\right)}{2 \cos \theta}
\]  

where \( \rho_o \) = density of oil, gm/cm\(^3\)

\( \sigma_{ow} \) = interfacial tension between the oil and the water, dyne/cm
CAPILLARY PRESSURE

The capillary forces in a petroleum reservoir are the result of the combined effect of the surface and interfacial tensions of the rock and fluids, the pore size and geometry, and the wetting characteristics of the system. Any curved surface between two immiscible fluids has the tendency to contract into the smallest possible area per unit volume. This is true whether the fluids are oil and water, water and gas (even air), or oil and gas. When two immiscible fluids are in contact, a discontinuity in pressure exists between the two fluids, which depends upon the curvature of the interface separating the fluids. We call this pressure difference the capillary pressure and it is referred to by $p_c$.

The displacement of one fluid by another in the pores of a porous medium is either aided or opposed by the surface forces of capillary pressure. As a consequence, in order to maintain a porous medium partially saturated with nonwetting fluid and while the medium is also exposed to wetting fluid, it is necessary to maintain the pressure of the nonwetting fluid at a value greater than that in the wetting fluid.

Denoting the pressure in the wetting fluid by $p_w$ and that in the nonwetting fluid by $p_{nw}$, the capillary pressure can be expressed as:

$$p_c = p_{nw} - p_w$$

That is, the pressure excess in the nonwetting fluid is the capillary pressure, and this quantity is a function of saturation. This is the defining equation for capillary pressure in a porous medium.

There are three types of capillary pressure:

- Water-oil capillary pressure (denoted as $P_{cwo}$)
- Gas-oil capillary pressure (denoted as $P_{cg0}$)
- Gas-water capillary pressure (denoted as $P_{cgw}$)

Applying the mathematical definition of the capillary pressure as expressed by Equation 4-24, the three types of the capillary pressure can be written as:

$$p_{cwo} = p_o - p_w$$
\[ p_{cgw} = p_{cg} - p_{cw} \]

where \( p_g, p_o, \) and \( p_w \) represent the pressure of gas, oil, and water, respectively.

If all the three phases are continuous, then:

\[ p_{cgw} = p_{cg} + p_{cw} \]

Referring to Figure 4-3, the pressure difference across the interface between Points 1 and 2 is essentially the capillary pressure, i.e.:

\[ p_c = p_1 - p_2 \quad (4-25) \]

The pressure of the water phase at Point 2 is equal to the pressure at point 4 minus the head of the water, or:

\[ p_2 = p_4 - gh \rho_w \quad (4-26) \]

The pressure just above the interface at Point 1 represents the pressure of the air and is given by:

\[ p_1 = p_3 - gh \rho_{air} \quad (4-27) \]

It should be noted that the pressure at Point 4 within the capillary tube is the same as that at Point 3 outside the tube. Subtracting Equation 4-26 from 4-27 gives:

\[ p_c = gh (\rho_w - \rho_{air}) = gh \Delta \rho \quad (4-28) \]

where \( \Delta \rho \) is the density difference between the wetting and nonwetting phase. The density of the air (gas) is negligible in comparison with the water density.

In practical units, Equation 4-28 can be expressed as:

\[ p_c = \left( \frac{h}{144} \right) \Delta \rho \]

where \( p_c \) = capillary pressure, psi

\( h \) = capillary rise, ft

\( \Delta \rho \) = density difference, lb/ft\(^3\)
In the case of an oil-water system, Equation 4-28 can be written as:

\[ p_c = gh (\rho_w - \rho_o) = gh \Delta \rho \]  

(4-29)

and in practical units

\[ p_c = \left( \frac{h}{144} \right) (\rho_w - \rho_o) \]

The capillary pressure equation can be expressed in terms of the surface and interfacial tension by combining Equations 4-28 and 4-29 with Equations 4-22 and 4-23 to give:

**• Gas-liquid system**

\[ p_c = \frac{2 \sigma_{gw} (\cos \theta)}{r} \]  

(4-30)

and

\[ h = \frac{2 \sigma_{gw} (\cos \theta)}{rg(\rho_w - \rho_{gas})} \]  

(4-31)

where \( \rho_w \) = water density, gm/cm\(^3\)
\( \sigma_{gw} \) = gas-water surface tension, dynes/cm
\( r \) = capillary radius, cm
\( \theta \) = contact angle
\( h \) = capillary rise, cm
\( g \) = acceleration due to gravity, cm/sec\(^2\)
\( p_c \) = capillary pressure, dynes/cm\(^2\)

**• Oil-water system**

\[ p_c = \frac{2 \sigma_{ow} (\cos \theta)}{r} \]  

(4-32)

and

\[ h = \frac{2 \sigma_{ow} (\cos \theta)}{rg(\rho_w - \rho_o)} \]  

(4-33)

where \( \sigma_{ow} \) is the water-oil interfacial tension.
Example 4-4

Calculate the pressure difference, i.e., capillary pressure, and capillary rise in an oil-water system from the following data:

\[
\begin{align*}
\theta &= 30^\circ \\
\rho_w &= 1.0 \text{ gm/cm}^3 \\
\rho_o &= 0.75 \text{ gm/cm}^3 \\
r &= 10^{-4} \text{ cm} \\
\sigma_{ow} &= 25 \text{ dynes/cm}
\end{align*}
\]

Solution

Step 1. Apply Equation 4-32 to give

\[
p_c = \frac{(2)(25)(\cos 30^\circ)}{0.0001} = 4.33 \times 10^5 \text{ dynes/cm}^2
\]

Since 1 dyne/cm\(^2\) = 1.45 \times 10^{-5} \text{ psi}, then

\[
p_c = 6.28 \text{ psi}
\]

This result indicates that the oil-phase pressure is 6.28 psi higher than the water-phase pressure.

Step 2. Calculate the capillary rise by applying Equation 4-33.

\[
h = \frac{(2)(25)(\cos 30^\circ)}{(0.0001)(980.7)(1.0 - 0.75)} = 1766 \text{ cm} = 75.9 \text{ ft}
\]

Capillary Pressure of Reservoir Rocks

The interfacial phenomena described above for a single capillary tube also exist when bundles of interconnected capillaries of varying sizes exist in a porous medium. The capillary pressure that exists within a porous medium between two immiscible phases is a function of the interfacial tensions and the average size of the capillaries which, in turn, controls the curvature of the interface. In addition, the curvature is also a function of the saturation distribution of the fluids involved.

Laboratory experiments have been developed to simulate the displacing forces in a reservoir in order to determine the magnitude of the capillary forces in a reservoir and, thereby, determine the fluid saturation dis-
tributions and connate water saturation. One such experiment is called the *restored capillary pressure technique* which was developed primarily to determine the magnitude of the connate water saturation. A diagrammatic sketch of this equipment is shown in Figure 4-4.

Briefly, this procedure consists of saturating a core 100% with the reservoir water and then placing the core on a porous membrane which is saturated 100% with water and is permeable to the water only, under the pressure drops imposed during the experiment. Air is then admitted into the core chamber and the pressure is increased until a small amount of water is displaced through the porous, semi-permeable membrane into the graduated cylinder. Pressure is held constant until no more water is displaced, which may require several days or even several weeks, after

![Figure 4-4. Capillary pressure equipment. (After Cole, F., 1969.)](image-url)
which the core is removed from the apparatus and the water saturation determined by weighing. The core is then replaced in the apparatus, the pressure is increased, and the procedure is repeated until the water saturation is reduced to a minimum.

The data from such an experiment are shown in Figure 4-5. Since the pressure required to displace the wetting phase from the core is exactly equal to the capillary forces holding the remaining water within the core after equilibrium has been reached, the pressure data can be plotted as capillary pressure data. Two important phenomena can be observed in Figure 4-5. First, there is a finite capillary pressure at 100% water saturation that is necessary to force the nonwetting phase into a capillary filled with the wetting phase. This minimum capillary pressure is known as the displacement pressure, $p_d$. 

![Figure 4-5. Capillary pressure curve.](image-url)
If the largest capillary opening is considered as circular with a radius of $r$, the pressure needed for forcing the nonwetting fluid out of the core is:

$$p_c = \frac{2 \sigma (\cos \theta)}{r}$$

This is the minimum pressure that is required to displace the wetting phase from the largest capillary pore because any capillary of smaller radius will require a higher pressure.

As the wetting phase is displaced, the second phenomenon of any immiscible displacement process is encountered, that is, the reaching of some finite minimum irreducible saturation. This irreducible water saturation is referred to as connate water.

It is possible from the capillary pressure curve to calculate the average size of the pores making up a stated fraction of the total pore space. Let $p_c$ be the average capillary pressure for the 10% between saturation of 40 and 50%. The average capillary radius is obtained from

$$r = \frac{2 \sigma (\cos \theta)}{p_c}$$

The above equation may be solved for $r$ providing that the interfacial tension $\sigma$, and the angle of contact $\theta$ may be evaluated.

Figure 4-6 is an example of typical oil-water capillary pressure curves. In this case, capillary pressure is plotted versus water saturation for four rock samples with permeabilities increasing from $k_1$ to $k_4$. It can be seen that, for decreases in permeability, there are corresponding increases in capillary pressure at a constant value of water saturation. This is a reflection of the influence of pore size since the smaller diameter pores will invariably have the lower permeabilities. Also, as would be expected the capillary pressure for any sample increases with decreasing water saturation, another indication of the effect of the radius of curvature of the water-oil interface.

**Capillary Hysteresis**

It is generally agreed that the pore spaces of reservoir rocks were originally filled with water, after which oil moved into the reservoir, displacing some of the water and reducing the water to some residual saturation. When discovered, the reservoir pore spaces are filled with a connate-water saturation and an oil saturation. All laboratory experiments are
designed to duplicate the saturation history of the reservoir. The process of generating the capillary pressure curve by displacing the wetting phase, i.e., water, with the nonwetting phase (such as with gas or oil), is called the drainage process.

This drainage process establishes the fluid saturations which are found when the reservoir is discovered. The other principal flow process of interest involves reversing the drainage process by displacing the nonwetting phase (such as with oil) with the wetting phase, (e.g., water). This displacing process is termed the imbibition process and the resulting curve is termed the capillary pressure imbibition curve. The process of saturating and desaturating a core with the nonwetting phase is called capillary hysteresis. Figure 4-7 shows typical drainage and imbibition
capillary pressure curves. The two capillary pressure-saturation curves are not the same.

This difference in the saturating and desaturating of the capillary-pressure curves is closely related to the fact that the advancing and receding contact angles of fluid interfaces on solids are different. Frequently, in natural crude oil-brine systems, the contact angle or wettability may change with time. Thus, if a rock sample that has been thoroughly cleaned with volatile solvents is exposed to crude oil for a period of time, it will behave as though it were oil wet. But if it is exposed to brine after...
cleaning, it will appear water wet. At the present time, one of the greatest unsolved problems in the petroleum industry is that of wettability of reservoir rock.

Another mechanism that has been proposed by McCardell (1955) to account for capillary hysteresis is called the *ink-bottle effect*. This phenomenon can be easily observed in a capillary tube having variations in radius along its length. Consider a capillary tube of axial symmetry having roughly sinusoidal variations in radius. When such a tube has its lower end immersed in water, the water will rise in the tube until the hydrostatic fluid head in the tube becomes equal to the capillary pressure. If then the tube is lifted to a higher level in the water, some water will drain out, establishing a new equilibrium level in the tube.

When the meniscus is advancing and it approaches a constriction, it *jumps* through the neck, whereas when receding, it halts without passing through the neck. This phenomenon explains why a given capillary pressure corresponds to a higher saturation on the drainage curve than on the imbibition curve.

### Initial Saturation Distribution in a Reservoir

An important application of the concept of capillary pressures pertains to the fluid distribution in a reservoir prior to its exploitation. The capillary pressure-saturation data can be converted into height-saturation data by arranging Equation 4-29 and solving for the height \( h \) above the free-water level.

\[
h = \frac{144 \ p_c}{\Delta \rho}
\]

(4 - 34)

where \( p_c \) = capillary pressure, psia
\( \Delta \rho \) = density difference between the wetting and nonwetting phase, lb/ft\(^3\)
\( H \) = height above the free-water level, ft

Figure 4-8 shows a plot of the water saturation distribution as a function of distance from the free-water level in an oil-water system.

It is essential at this point to introduce and define four important concepts:

- Transition zone
- Water-oil contact (WOC)
- Gas-oil contact (GOC)
- Free water level (FWL)
Figure 4-9 illustrates an idealized gas, oil, and water distribution in a reservoir. The figure indicates that the saturations are gradually charging from 100% water in the water zone to irreducible water saturation some vertical distance above the water zone. This vertical area is referred to as the transition zone, which must exist in any reservoir where there is a bottom water table. The transition zone is then defined as the vertical thickness over which the water saturation ranges from 100% saturation to irreducible water saturation $S_{wc}$. The important concept to be gained from Figure 4-9 is that there is no abrupt change from 100% water to maximum oil saturation. The creation of the oil-water transition zone is one of the major effects of capillary forces in a petroleum reservoir.

Similarly, the total liquid saturation (i.e. oil and water) is smoothly changing from 100% in the oil zone to the connate water saturation in the gas cap zone. A similar transition exists between the oil and gas zone. Figure 4-8 serves as a definition of what is meant by gas-oil and water-oil contacts. The WOC is defined as the “uppermost depth in the reservoir where a 100% water saturation exists.” The GOC is defined as the “mini-
mum depth at which a 100% liquid, i.e., oil + water, saturation exists in the reservoir.”

Section A of Figure 4-10 shows a schematic illustration of a core that is represented by five different pore sizes and completely saturated with water, i.e., wetting phase. Assume that we subject the core to oil (the nonwetting phase) with increasing pressure until some water is displaced from the core, i.e., displacement pressure $p_d$. This water displacement will occur from the largest pore size. The oil pressure will have to increase to displace the water in the second largest pore. This sequential process is shown in sections B and C of Figure 4-10.

It should be noted that there is a difference between the free water level (FWL) and the depth at which 100% water saturation exists. From a reservoir engineering standpoint, the free water level is defined by zero capillary pressure. Obviously, if the largest pore is so large that there is no capillary rise in this size pore, then the free water level and 100% water saturation level, i.e., WOC, will be the same. This concept can be expressed mathematically by the following relationship:

$$FWL = WOC + \frac{144 p_d}{\Delta \rho}$$  \hspace{1cm} (4-35)
where \( p_d \) = displacement pressure, psi
\( \Delta \rho \) = density difference, lb/ft\(^3\)
FWL = free water level, ft
WOC = water-oil contact, ft

**Example 4-5**

The reservoir capillary pressure-saturation data of the Big Butte Oil reservoir is shown graphically in Figure 4-11. Geophysical log interpretations and core analysis establish the WOC at 5023 ft. The following additional data are available:

- Oil density = 43.5 lb/ft\(^3\)
- Water density = 64.1 lb/ft\(^3\)
- Interfacial tension = 50 dynes/cm

Calculate:

- Connate water saturation (\( S_{wc} \))
- Depth to FWL
- Thickness of the transition zone
- Depth to reach 50% water saturation

---

**Figure 4-10.** Relationship between saturation profile and pore-size distribution.
Solution

a. From Figure 4-11, connate-water saturation is 20%.

b. Applying Equation 4-35 with a displacement pressure of 1.5 psi gives

\[
FWL = 5023 + \frac{(144)(1.5)}{(64.1 - 43.5)} = 5033.5 \text{ ft}
\]

c. Thickness of transition zone = \( \frac{144(6.0 - 1.5)}{(64.1 - 43.5)} = 31.5 \text{ ft} \)

d. \( P_c \) at 50% water saturation = 3.5 psia

   Equivalent height above the FWL = \( (144)(3.5)/(64.1 - 43.25) = 24.5 \text{ ft} \)
   Depth to 50% water saturation = 5033.5 - 24.5 = 5009 ft

The above example indicates that only oil will flow in the interval between the top of the pay zone and depth of 4991.5 ft. In the transition zone, i.e., the interval from 4991.5 ft to the WOC, oil production would be accompanied by simultaneous water production.

It should be pointed out that the thickness of the transition zone may range from few feet to several hundred feet in some reservoirs. Recalling the capillary rise equation, i.e., height above FWL,
The above expression shows that as the radius of the pore $r$ increases the volume of $h$ decreases. Therefore, a reservoir rock system with small pore sizes will have a longer transition zone than a reservoir rock system comprised of large pore sizes.

$$h = \frac{2 \sigma (\cos \phi)}{r g \Delta \rho}$$

The above relationship suggests that the height above FWL increases with decreasing the density difference $\Delta \rho$.

From a practical standpoint, this means that in a gas reservoir having a gas-water contact, the thickness of the transition zone will be a minimum since $\Delta \rho$ will be large. Also, if all other factors remain unchanged, a low API gravity oil reservoir with an oil-water contact will have a longer transition zone than a high API gravity oil reservoir. Cole (1969) illustrated this concept graphically in Figure 4-12.

The above expression also shows that as the radius of the pore $r$ increases the volume of $h$ decreases. Therefore, a reservoir rock system with small pore sizes will have a longer transition zone than a reservoir rock system comprised of large pore sizes.

Figure 4-12. Variation of transition zone with fluid gravity. (After Cole, F., 1969.)
The reservoir pore size can often be related approximately to permeability, and where this applies, it can be stated that high permeability reservoirs will have shorter transition zones than low permeability reservoirs as shown graphically in Figure 4-13. As shown by Cole (Figure 4-14), a tilted water-oil contact could be caused by a change in permeability across the reservoir. It should be emphasized that the factor responsible for this change in the location of the water-oil contact is actually a change in the size of the pores in the reservoir rock system.

The previous discussion of capillary forces in reservoir rocks has assumed that the reservoir pore sizes, i.e., permeabilities, are essentially uniform. Cole (1969) discussed the effect of reservoir non-uniformity on the distribution of the fluid saturation through the formation. Figure 4-15 shows a hypothetical reservoir rock system that is comprised of seven layers. In addition, the seven layers are characterized by only two different pore sizes, i.e., permeabilities, and corresponding capillary pressure

\[ p_c = \text{medium} \]

\[ p_c = \text{low} \]

\[ p_c = \text{high} \]

\[ \text{Water saturation, } \% \]

\[ 0 \quad 100 \]

**Figure 4-13.** Variation of transition zone with permeability.
curves as shown in section A of Figure 4-15. The resulting capillary pressure curve for the layered reservoir would resemble that shown in section B of Figure 4-15. If a well were drilled at the point shown in section B of Figure 4-15, Layers 1 and 3 would not produce water, while Layer 2, which is above Layer 3, would produce water since it is located in the transition zone.

**Example 4-6**

A four-layer oil reservoir is characterized by a set of reservoir capillary pressure-saturation curves as shown in Figure 4-16. The following additional data are also available.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth, ft</th>
<th>Permeability, md</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4000–4010</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>4010–4020</td>
<td>190</td>
</tr>
<tr>
<td>3</td>
<td>4020–4035</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>4035–4060</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 4-15. Effect of permeability on water saturation profile. (After Cole, F., 1969.)
Calculate and plot water saturation versus depth for this reservoir.

**Solution**

*Step 1.* Establish the FWL by determining the displacement pressure $p_d$ for the bottom layer, i.e., Layer 4, and apply Equation 4-37:

$\bullet$ $p_d = 0.75$ psi

$$FWL = 4060 + \frac{(144)(0.75)}{(65.2 - 55.2)} = 4070.8 \text{ ft}$$

*Step 2.* The top of the bottom layer is located at a depth of 4035 ft which is 35.8 ft above the FWL. Using that height $h$ of 35.8 ft, calculate the capillary pressure at the top of the bottom layer.

$\bullet$ $p_c = \left( \frac{h}{144} \right) \Delta \rho = \left( \frac{35.8}{144} \right) (65.2 - 55.2) = 2.486 \text{ psi}$
• From the capillary pressure-saturation curve designated for Layer 4, read the water saturation that corresponds to a \( p_c \) of 2.486 to give \( S_w = 0.23 \).

• Assume different values of water saturations and convert the corresponding capillary pressures into height above the FWL by applying Equation 4-34.

\[
h = \frac{144 \, p_c}{\rho_w - \rho_o}
\]

<table>
<thead>
<tr>
<th>( S_w )</th>
<th>( p_c, \text{ psi} )</th>
<th>( h, \text{ ft} )</th>
<th>Depth = FWL − h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23</td>
<td>2.486</td>
<td>35.8</td>
<td>4035</td>
</tr>
<tr>
<td>0.25</td>
<td>2.350</td>
<td>33.84</td>
<td>4037</td>
</tr>
<tr>
<td>0.30</td>
<td>2.150</td>
<td>30.96</td>
<td>4040</td>
</tr>
<tr>
<td>0.40</td>
<td>1.800</td>
<td>25.92</td>
<td>4045</td>
</tr>
<tr>
<td>0.50</td>
<td>1.530</td>
<td>22.03</td>
<td>4049</td>
</tr>
<tr>
<td>0.60</td>
<td>1.340</td>
<td>19.30</td>
<td>4052</td>
</tr>
<tr>
<td>0.70</td>
<td>1.200</td>
<td>17.28</td>
<td>4054</td>
</tr>
<tr>
<td>0.80</td>
<td>1.050</td>
<td>15.12</td>
<td>4056</td>
</tr>
<tr>
<td>0.90</td>
<td>0.900</td>
<td>12.96</td>
<td>4058</td>
</tr>
</tbody>
</table>

Step 3. The top of Layer 3 is located at a distance of 50.8 ft from the FWL (i.e., \( h = 4070.8 - 4020 = 50.8 \) ft). Calculate the capillary pressure at the top of the third layer:

\[
p_c = \left( \frac{50.8}{144} \right) (65.2 - 55.2) = 3.53 \text{ psi}
\]

• The corresponding water saturation as read from the curve designated for Layer 3 is 0.370.

• Construct the following table for Layer 3.

<table>
<thead>
<tr>
<th>( S_w )</th>
<th>( p_c, \text{ psi} )</th>
<th>( h, \text{ ft} )</th>
<th>Depth = FWL − h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>3.53</td>
<td>50.8</td>
<td>4020</td>
</tr>
<tr>
<td>0.40</td>
<td>3.35</td>
<td>48.2</td>
<td>4023</td>
</tr>
<tr>
<td>0.50</td>
<td>2.75</td>
<td>39.6</td>
<td>4031</td>
</tr>
<tr>
<td>0.60</td>
<td>2.50</td>
<td>36.0</td>
<td>4035</td>
</tr>
</tbody>
</table>
Step 4. • Distance from the FWL to the top of Layer 2 is:
\[ h = 4070.8 - 4010 = 60.8 \text{ ft} \]
• \( p_c = \left( \frac{60.8}{144} \right) (65.2 - 55.2) = 4.22 \text{ psi} \)
• \( S_w \) at \( p_c \) of 4.22 psi is 0.15.
• Distance from the FWL to the bottom of the layer is 50.8 ft that corresponds to a \( p_c \) of 3.53 psi and \( S_w \) of 0.15. This indicates that the second layer has a uniform water saturation of 15%.

Step 5. For Layer 1, distance from the FWL to the top of the layer:
• \( h = 4070.8 - 4000 = 70.8 \text{ ft} \)
• \( p_c = \left( \frac{70.8}{144} \right) (10) = 4.92 \text{ psi} \)
• \( S_w \) at the top of Layer 1 = 0.25
• The capillary pressure at the bottom of the layer is 3.53 psi with a corresponding water saturation of 0.27.

Step 6. Figure 4-17 documents the calculated results graphically. The figure indicates that Layer 2 will produce 100% oil while all remaining layers produce oil and water simultaneously.

![Figure 4-17. Water saturation profile.](image-url)
Leverett J-Function

Capillary pressure data are obtained on small core samples that represent an extremely small part of the reservoir and, therefore, it is necessary to combine all capillary data to classify a particular reservoir. The fact that the capillary pressure-saturation curves of nearly all naturally porous materials have many features in common has led to attempts to devise some general equation describing all such curves. Leverett (1941) approached the problem from the standpoint of dimensional analysis.

Realizing that capillary pressure should depend on the porosity, interfacial tension, and mean pore radius, Leverett defined the dimensionless function of saturation, which he called the J-function, as

\[
J(S_w) = 0.21645 \frac{p_c}{\sigma} \left(\frac{k}{\phi}\right)^{1/2}
\]

where
- \( J(S_w) \) = Leverett J-function
- \( p_c \) = capillary pressure, psi
- \( \sigma \) = interfacial tension, dynes/cm
- \( k \) = permeability, md
- \( \phi \) = fractional porosity

In doing so, Leverett interpreted the ratio of permeability, \( k \), to porosity, \( \phi \), as being proportional to the square of a mean pore radius.

The J-function was originally proposed as a means of converting all capillary-pressure data to a universal curve. There are significant differences in correlation of the J-function with water saturation from formation to formation, so that no universal curve can be obtained. For the same formation, however, this dimensionless capillary-pressure function serves quite well in many cases to remove discrepancies in the \( p_c \) versus \( S_w \) curves and reduce them to a common curve. This is shown for various unconsolidated sands in Figure 4-18.

Example 4-7

A laboratory capillary pressure test was conducted on a core sample taken from the Nameless Field. The core has a porosity and permeability of 16% and 80 md, respectively. The capillary pressure-saturation data are given as follows:
<table>
<thead>
<tr>
<th>$S_w$</th>
<th>$p_c$, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td>0.8</td>
<td>0.60</td>
</tr>
<tr>
<td>0.6</td>
<td>0.75</td>
</tr>
<tr>
<td>0.4</td>
<td>1.05</td>
</tr>
<tr>
<td>0.2</td>
<td>1.75</td>
</tr>
</tbody>
</table>

**Figure 4-18.** The Leverett $J$-function for unconsolidated sands. (After Leverett, 1941.)
The interfacial tension is measured at 50 dynes/cm. Further reservoir engineering analysis indicated that the reservoir is better described at a porosity value of 19% and an absolute permeability of 120 md. Generate the capillary pressure data for the reservoir.

**Solution**

**Step 1.** Calculate the J-function using the measured capillary pressure data.

\[
J(S_w) = 0.21645 \left( \frac{p_c}{50} \right)^{\frac{1.0}{0.16}} = 0.096799 \ p_c
\]

<table>
<thead>
<tr>
<th>( S_w )</th>
<th>( p_c )</th>
<th>( J(S_w) = 0.096799 \ p_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.50</td>
<td>0.048</td>
</tr>
<tr>
<td>0.8</td>
<td>0.60</td>
<td>0.058</td>
</tr>
<tr>
<td>0.6</td>
<td>0.75</td>
<td>0.073</td>
</tr>
<tr>
<td>0.4</td>
<td>1.05</td>
<td>0.102</td>
</tr>
<tr>
<td>0.2</td>
<td>1.75</td>
<td>0.169</td>
</tr>
</tbody>
</table>

**Step 2.** Using the new porosity and permeability values, solve Equation 4-36 for the capillary pressure \( p_c \).

\[
p_c = J(S_w) \sigma \left[ \frac{0.21645 \frac{k}{\sqrt{\phi}}}{0.21645 \frac{120}{0.19}} \right]
\]

\[
p_c = J(S_w) \left( \frac{50}{0.21645 \frac{120}{0.19}} \right)
\]

\[
p_c = 9.192 J(S_w)
\]

**Step 3.** Reconstruct the capillary pressure-saturation table.

<table>
<thead>
<tr>
<th>( S_w )</th>
<th>( J(S_w) )</th>
<th>( p_c = 9.192 J(S_w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.048</td>
<td>0.441</td>
</tr>
<tr>
<td>0.8</td>
<td>0.058</td>
<td>0.533</td>
</tr>
<tr>
<td>0.6</td>
<td>0.073</td>
<td>0.671</td>
</tr>
<tr>
<td>0.4</td>
<td>0.102</td>
<td>0.938</td>
</tr>
<tr>
<td>0.2</td>
<td>0.169</td>
<td>1.553</td>
</tr>
</tbody>
</table>
Converting Laboratory Capillary Pressure Data

For experimental convenience, it is common in the laboratory determination of capillary pressure to use air-mercury or air-brine systems, rather than the actual water-oil system characteristic of the reservoir. Since the laboratory fluid system does not have the same surface tension as the reservoir system, it becomes necessary to convert laboratory capillary pressure to reservoir capillary pressure. By assuming that the Leverett J-function is a property of rock and does not change from the laboratory to the reservoir, we can calculate reservoir capillary pressure as shown below.

\[(p_c)_{res} = (p_c)_{lab} \frac{\sigma_{res}}{\sigma_{lab}}\]  

Even after the laboratory capillary pressure has been corrected for surface tension, it may be necessary to make further corrections for permeability and porosity. The reason for this is that the core sample that was used in performing the laboratory capillary pressure test may not be representative of the average reservoir permeability and porosity. If we assume that the J-function will be invariant for a given rock type over a range of porosity and permeability values, then the reservoir capillary pressure can be expressed as

\[(p_c)_{res} = (p_c)_{lab} \frac{\sigma_{res}}{\sigma_{lab}} \sqrt{(\phi_{res} k_{core}) / (\phi_{core} k_{res})} \]  

where
- \((p_c)_{res}\) = reservoir capillary pressure
- \(\sigma_{res}\) = reservoir surface or interfacial tension
- \(k_{res}\) = reservoir permeability
- \(\phi_{res}\) = reservoir porosity
- \((p_c)_{lab}\) = laboratory measured capillary pressure
- \(\phi_{core}\) = core porosity
- \(k_{core}\) = core permeability

**PERMEABILITY**

Permeability is a property of the porous medium that measures the capacity and ability of the formation to transmit fluids. The rock permeability, \(k\), is a very important rock property because it controls the direc-
tional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by Henry Darcy in 1856. In fact, the equation that defines permeability in terms of measurable quantities is called Darcy’s Law.

Darcy developed a fluid flow equation that has since become one of the standard mathematical tools of the petroleum engineer. If a horizontal linear flow of an incompressible fluid is established through a core sample of length L and a cross-section of area A, then the governing fluid flow equation is defined as

\[ n = \frac{k}{\mu} \frac{dp}{dL} \]  

(4-38)

where
- \( n \) = apparent fluid flowing velocity, cm/sec
- \( k \) = proportionality constant, or permeability, Darcys
- \( \mu \) = viscosity of the flowing fluid, cp
- \( \frac{dp}{dL} \) = pressure drop per unit length, atm/cm

The velocity, \( n \), in Equation 4-38 is not the actual velocity of the flowing fluid but is the apparent velocity determined by dividing the flow rate by the cross-sectional area across which fluid is flowing. Substituting the relationship, \( q/A \), in place of \( n \) in Equation 4-38 and solving for \( q \) results in

\[ q = -\frac{kA}{\mu} \frac{dp}{dL} \]  

(4-39)

where
- \( q \) = flow rate through the porous medium, cm³/sec
- \( A \) = cross-sectional area across which flow occurs, cm²

With a flow rate of one cubic centimeter per second across a cross-sectional area of one square centimeter with a fluid of one centipoise viscosity and a pressure gradient at one atmosphere per centimeter of length, it is obvious that \( k \) is unity. For the units described above, \( k \) has been arbitrarily assigned a unit called Darcy in honor of the man responsible for the development of the theory of flow through porous media. Thus, when all other parts of Equation 4-39 have values of unity, \( k \) has a value of one Darcy.

One Darcy is a relatively high permeability as the permeabilities of most reservoir rocks are less than one Darcy. In order to avoid the use of
fractions in describing permeabilities, the term *millidarcy* is used. As the term indicates, one millidarcy, i.e., 1 md, is equal to one-thousandth of one Darcy or,

\[ 1 \text{ Darcy} = 1000 \text{ md} \]

The negative sign in Equation 4-39 is necessary as the pressure increases in one direction while the length increases in the opposite direction.

Equation 4-39 can be integrated when the geometry of the system through which fluid flows is known. For the simple linear system shown in Figure 4-19, the integration is performed as follows:

\[
q \int_0^L dL = -\frac{kA}{\mu} \int_{p_1}^{p_2} dp
\]

Integrating the above expression yields:

\[
qL = -\frac{kA}{\mu} (p_2 - p_1)
\]

*Figure 4-19.* Linear flow model.
It should be pointed out that the volumetric flow rate, q, is constant for liquids because the density does not change significantly with pressure.

Since \( p_1 \) is greater than \( p_2 \), the pressure terms can be rearranged, which will eliminate the negative term in the equation. The resulting equation is:

\[
q = \frac{kA (p_1 - p_2)}{\mu L} \tag{4-40}
\]

Equation 4-40 is the conventional linear flow equation used in fluid flow calculations.

Standard laboratory analysis procedures will generally provide reliable data on permeability of core samples. If the rock is not homogeneous, the whole core analysis technique will probably yield more accurate results than the analysis of core plugs (small pieces cut from the core). Procedures that have been used for improving the accuracy of the permeability determination include cutting the core with an oil-base mud, employing a pressure-core barrel, and conducting the permeability tests with reservoir oil.

Permeability is reduced by overburden pressure, and this factor should be considered in estimating permeability of the reservoir rock in deep wells because permeability is an isotropic property of porous rock in some defined regions of the system, that is, it is directional. Routine core analysis is generally concerned with plug samples drilled parallel to bedding planes and, hence, parallel to direction of flow in the reservoir. These yield horizontal permeabilities (\( k_h \)).

The measured permeability on plugs that are drilled perpendicular to bedding planes are referred to as vertical permeability (\( k_v \)). Figure 4-20 shows a schematic illustration of the concept of the core plug and the associated permeability.

As shown in Figure 4-20, there are several factors that must be considered as possible sources of error in determining reservoir permeability. These factors are:

1. Core sample may not be representative of the reservoir rock because of reservoir heterogeneity.
2. Core recovery may be incomplete.
3. Permeability of the core may be altered when it is cut, or when it is cleaned and dried in preparation for analysis. This problem is likely to occur when the rock contains reactive clays.
4. Sampling process may be biased. There is a temptation to select the best parts of the core for analysis.
Permeability is measured by passing a fluid of known viscosity $\mu$ through a core plug of measured dimensions ($A$ and $L$) and then measuring flow rate $q$ and pressure drop $\Delta p$. Solving Equation 4-40 for the permeability, gives:

$$k = \frac{q \mu L}{A \Delta p}$$
where \( L \) = length of core, cm  
\( A \) = cross-sectional area, cm\(^2\)

The following conditions must exist during the measurement of permeability:

- Laminar (viscous) flow
- No reaction between fluid and rock
- Only single phase present at 100% pore space saturation

This measured permeability at 100% saturation of a single phase is called the \textit{absolute permeability} of the rock.

**Example 4-8**

A brine is used to measure the absolute permeability of a core plug. The rock sample is 4 cm long and 3 cm\(^2\) in cross section. The brine has a viscosity of 1.0 cp and is flowing a constant rate of 0.5 cm\(^3\)/sec under a 2.0 atm pressure differential. Calculate the absolute permeability.

**Solution**

Applying Darcy’s equation, i.e., Equation 4-40, gives:

\[
0.5 = \left( \frac{k}{1} \right) \left( \frac{3}{4} \right) \left( \frac{2}{1} \right)
\]

\[ k = 0.333 \text{ Darcys} \]

**Example 4-9**

Rework the above example assuming that an oil of 2.0 cp is used to measure the permeability. Under the same differential pressure, the flow rate is 0.25 cm\(^3\)/sec.
Solution

Applying Darcy’s equation yields:

\[ 0.25 = \frac{(k)(3)(2)}{(2)(4)} \]

\[ k = 0.333 \text{ Darcys} \]

Dry gas is usually used (air, N\textsubscript{2}, He) in permeability determination because of its convenience, availability, and to minimize fluid-rock reaction.

The measurement of the permeability should be restricted to the low (laminar/viscous) flow rate region, where the pressure remains proportional to flow rate within the experimental error. For high flow rates, Darcy’s equation as expressed by Equation 4-40 is inappropriate to describe the relationship of flow rate and pressure drop.

In using dry gas in measuring the permeability, the gas volumetric flow rate \( q \) varies with pressure because the gas is a highly compressible fluid. Therefore, the value of \( q \) at the average pressure in the core must be used in Equation 4-40. Assuming the used gases follow the ideal gas behavior (at low pressures), the following relationships apply:

\[ p_1 V_1 = p_2 V_2 = p_m V_m \]

In terms of the flow rate \( q \), the above equation can be equivalently expressed as:

\[ p_1 q_1 = p_2 q_2 = p_m q_m \quad (4-41) \]

with the mean pressure \( p_m \) expressed as:

\[ p_m = \frac{p_1 + p_2}{2} \]

where \( p_1, p_2, p_m = \text{inlet, outlet, and mean pressures, respectively, atm} \)

\( V_1, V_2, V_m = \text{inlet, outlet, and mean gas volume, respectively, cm}^3 \)

\( q_1, q_2, q_m = \text{inlet, outlet, and mean gas flow rate, respectively, cm}^3/\text{sec} \)
The gas flow rate is usually measured at base (atmospheric) pressure $p_b$ and, therefore, the term $Q_{gsc}$ is introduced into Equation 4-41 to produce:

$$Q_{gsc} \ p_b = q_m \ p_m$$

where $Q_{gsc} =$ gas flow rate at standard conditions, cm$^3$/sec

$p_b =$ base pressure (atmospheric pressure), atm

Substituting Darcy’s Law in the above expression gives

$$Q_{gsc} \ p_b = \frac{k \ A \ (p_1 - p_2)}{\mu_g \ L} \left( p_1 + p_2 \right)$$

or

$$Q_{gsc} = \frac{k \ A \ (p_1^2 - p_2^2)}{2 \ \mu_g \ L \ p_b} \quad (4 - 42)$$

where $k =$ absolute permeability, Darcys

$\mu_g =$ gas viscosity, cp

$p_b =$ base pressure, atm

$p_1 =$ inlet (upstream) pressure, atm

$p_2 =$ outlet (downstream) pressure, atm

$L =$ length of the core, cm

$A =$ cross-sectional area, cm$^2$

$Q_{gsc} =$ gas flow rate at standard conditions, cm$^3$/sec

The Klinkenberg Effect

Klinkenberg (1941) discovered that permeability measurements made with air as the flowing fluid showed different results from permeability measurements made with a liquid as the flowing fluid. The permeability of a core sample measured by flowing air is always greater than the permeability obtained when a liquid is the flowing fluid. Klinkenberg postulated, on the basis of his laboratory experiments, that liquids had a zero velocity at the sand grain surface, while gases exhibited some finite velocity at the sand grain surface. In other words, the gases exhibited slippage at the sand grain surface. This slippage resulted in a higher flow rate for the gas at a given pressure differential. Klinkenberg also found that for a given porous medium as the mean pressure increased the calculated permeability decreased.
Mean pressure is defined as upstream flowing plus downstream flowing pressure divided by two, \( p_m = (p_1 + p_2)/2 \). If a plot of measured permeability versus \( 1/p_m \) were extrapolated to the point where \( 1/p_m = 0 \), in other words, where \( p_m = \) infinity, this permeability would be approximately equal to the liquid permeability. A graph of this nature is shown in Figure 4-21. The absolute permeability is determined by extrapolation as shown in Figure 4-21.

The magnitude of the Klinkenberg effect varies with the core permeability and the type of the gas used in the experiment as shown in Figures 4-22 and 4-23. The resulting straight-line relationship can be expressed as

\[
k_g = k_L + c \left[ \frac{1}{p_m} \right]
\]

where

- \( k_g \) = measured gas permeability
- \( p_m \) = mean pressure
- \( k_L \) = equivalent liquid permeability, i.e., absolute permeability, \( k_c \)
- \( c \) = slope of the line

*Figure 4-21. The Klinkenberg effect in gas permeability measurements.*
Klinkenberg suggested that the slope $Ac$ is a function of the following factors:

- Absolute permeability $k$, i.e., permeability of medium to a single phase completely filling the pores of the medium $k_L$.
- Type of the gas used in measuring the permeability, e.g., air.
- Average radius of the rock capillaries.

Klinkenberg expressed the slope $c$ by the following relationship:

$$c = bk_L$$  \hspace{1cm} (4-44)$$

where $k_L$ = equivalent liquid permeability, i.e., absolute permeability, $k$

$b$ = constant which depends on the size of the pore openings and is inversely proportional to radius of capillaries.
Combining Equation 4-44 with 4-43 gives:

\[ k_g = k_L + (b \cdot k_L) \left( \frac{1}{p_m} \right) \]  \hspace{1cm} (4-45)

where \( k_g \) is the gas permeability as measured at the average pressure \( p_m \).

Jones (1972) studied the gas slip phenomena for a group of cores for which porosity, liquid permeability \( k_L \) (absolute permeability), and air permeability were determined. He correlated the parameter \( b \) with the liquid permeability by the following expression:

\[ b = 6.9 k_L^{-0.36} \]  \hspace{1cm} (4-46)

The usual measurement of permeability is made with air at mean pressure just above atmospheric pressure (1 atm). To evaluate the slip phenomenon and the Klinkenberg effect, it is necessary to at least measure the gas permeability at two mean-pressure levels. In the absence of such data, Equations 4-45 and 4-46 can be combined and arranged to give:

\[ 6.9 k_L^{0.64} + p_m k_L - p_m k_g = 0 \]  \hspace{1cm} (4-47)
where \( p_m \) = mean pressure, psi
\( k_g \) = air permeability at \( p_m \), psi
\( k_L \) = absolute permeability (k), md

Equation 4-47 can be used to calculate the absolute permeability when only one gas permeability measurement \( (k_g) \) of a core sample is made at \( p_m \). This nonlinear equation can be solved iteratively by using the Newton-Raphson iterative methods. The proposed solution method can be conveniently written as

\[
k_{i+1} = k_i - \frac{f(k_i)}{f'(k_i)}
\]

where \( k_i \) = initial guess of the absolute permeability, md
\( k_{i+1} \) = new permeability value to be used for the next iteration
\( i \) = iteration level
\( f(k_i) \) = Equation 4-47 as evaluated by using the assumed value of \( k_i \).
\( f'(k_i) \) = first-derivative of Equation 4-47 as evaluated at \( k_i \)

The first derivative of Equation 4-47 with respect to \( k_i \) is:

\[
f'(k_i) = 4.416 k_i^{-0.36} + p_m
\]  \hspace{1cm} (4-48)

The iterative procedure is repeated until convergence is achieved when \( f(k_i) \) approaches zero or when no changes in the calculated values of \( k_i \) are observed.

**Example 4-10**

The permeability of a core plug is measured by air. Only one measurement is made at a mean pressure of 2.152 psi. The air permeability is 46.6 md. Estimate the absolute permeability of the core sample. Compare the result with the actual absolute permeability of 23.66 md.

**Solution**

**Step 1.** Substitute the given values of \( p_m \) and \( k_g \) into Equations 4-47 and 4-48, to give:

\[
f(k_i) = 6.9 k_i^{0.64} + 2.152 k_i - (2.152) (46.6) f'(k_i) = 4.416 k_i^{-0.36} + 2.152
\]

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Step 2. Assume $k_i = 30$ and apply the Newton-Raphson method to find the required solution as shown below.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$k_i$</th>
<th>$f(k_i)$</th>
<th>$f'(k_i)$</th>
<th>$k_{i+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.000</td>
<td>25.12</td>
<td>3.45</td>
<td>22.719</td>
</tr>
<tr>
<td>2</td>
<td>22.719</td>
<td>-0.466</td>
<td>3.29</td>
<td>22.861</td>
</tr>
<tr>
<td>3</td>
<td>22.861</td>
<td>0.414</td>
<td>3.29</td>
<td>22.848</td>
</tr>
</tbody>
</table>

After three iterations, the Newton-Raphson method converges to an absolute value for the permeability of 22.848 md.

Equation 4-39 can be expanded to describe flow in any porous medium where the geometry of the system is not too complex to integrate. For example, the flow into a well bore is not linear, but is more often radial. Figure 4-24 illustrates the type of flow that is typical of that occurring in the vicinity of a producing well. For a radial flow, Darcy’s equation in a differential form can be written as:

![Figure 4-24. Radial flow model.](image-url)
Integrating Darcy's equation gives:

\[ q = \frac{kA}{\mu} \int dp \]

The term \( dL \) has been replaced by \( dr \) as the length term has now become a radius term. The minus sign is no longer required for the radial system shown in Figure 4-24 as the radius increases in the same direction as the pressure. In other words, as the radius increases going away from the well bore, the pressure also increases. At any point in the reservoir, the cross-sectional area across which flow occurs will be the surface area of a cylinder, which is \( 2\pi rh \). Since the cross-sectional area is related to \( r \), then \( A \) must be included within the integral sign as follows:

\[ q = \frac{kA}{\mu} \int dp \]

**Figure 4-25.** Linear flow through layered beds.
Solving for the flow rate, $q$, results in:

The above equation assumes that the reservoir is homogeneous and is completely saturated with a single liquid phase (appropriate modifications will be discussed in later sections to account for the presence of other fluids), where:

$q = \text{flow rate, reservoir cm}^3/\text{sec}$  
$k = \text{absolute permeability, Darcy}$  
$h = \text{thickness, cm}$  
$r_c = \text{drainage radius, cm}$  
$r_w = \text{well bore radius, cm}$  
$p_c = \text{pressure at drainage radius, atm}$  
$p_{wf} = \text{bottom-hole flowing pressure}$  
$\mu = \text{viscosity, cp}$

**Averaging Absolute Permeabilities**

The most difficult reservoir properties to determine usually are the level and distribution of the absolute permeability throughout the reservoir. They are more variable than porosity and more difficult to measure. Yet an adequate knowledge of permeability distribution is critical to the
prediction of reservoir depletion by any recovery process. It is rare to encounter a homogeneous reservoir in actual practice. In many cases, the reservoir contains distinct layers, blocks, or concentric rings of varying permeabilities. Also, because smaller-scale heterogeneities always exist, core permeabilities must be averaged to represent the flow characteristics of the entire reservoir or individual reservoir layers (units). The proper way of averaging the permeability data depends on how permeabilities were distributed as the rock was deposited.

There are three simple permeability-averaging techniques that are commonly used to determine an appropriate average permeability to represent an equivalent homogeneous system. These are:

- Weighted-average permeability
- Harmonic-average permeability
- Geometric-average permeability

Weighted-Average Permeability

This averaging method is used to determine the average permeability of layered-parallel beds with different permeabilities. Consider the case where the flow system is comprised of three parallel layers that are separated from one another by thin impermeable barriers, i.e., no cross flow, as shown in Figure 4-25. All the layers have the same width w with a cross-sectional area of A.

The flow from each layer can be calculated by applying Darcy’s equation in a linear form as expressed by Equation 4-40, to give:

**Layer 1**

\[ q_1 = \frac{k_1 w h_1 \Delta p}{\mu L} \]

**Layer 2**

\[ q_2 = \frac{k_2 w h_2 \Delta p}{\mu L} \]

**Layer 3**

\[ q_3 = \frac{k_3 w h_3 \Delta p}{\mu L} \]
The total flow rate from the entire system is expressed as

\[ q_t = \frac{k_{\text{avg}} w h_t \Delta p}{\mu L} \]

where \( q_t \) = total flow rate
\( k_{\text{avg}} \) = average permeability for the entire model
\( w \) = width of the formation
\( \Delta p = p_1 - p_2 \)
\( h_t \) = total thickness

The total flow rate \( q_t \) is equal to the sum of the flow rates through each layer or:

\[ q_t = q_1 + q_2 + q_3 \]

Combining the above expressions gives:

\[ \frac{k_{\text{avg}} w h_t \Delta p}{\mu L} = \frac{k_1 w h_1 \Delta p}{\mu L} + \frac{k_2 w h_2 \Delta p}{\mu L} + \frac{k_3 w h_3 \Delta p}{\mu L} \]

or

\[ k_{\text{avg}} h_t = k_1 h_1 + k_2 h_2 + k_3 h_3 \]

\[ k_{\text{avg}} = \frac{k_1 h_1 + k_2 h_2 + k_3 h_3}{h_t} \]

The average absolute permeability for a parallel-layered system can be expressed in the following form:

\[ k_{\text{avg}} = \frac{\sum_{j=1}^{n} k_j h_j}{\sum_{j=1}^{n} h_j} \quad (4 - 50) \]

Equation 4-50 is commonly used to determine the average permeability of a reservoir from core analysis data.
Figure 4-26 shows a similar layered system with variable layers width. Assuming no cross-flow between the layers, the average permeability can be approximated in a manner similar to the above derivation to give:

\[
k_{\text{avg}} = \frac{\sum_{j=1}^{n} k_j A_j}{\sum_{j=1}^{n} A_j}
\]  

(4-51)

with

\[A_j = h_j w_j\]

where \(A_j\) = cross-sectional area of layer \(j\)
\(w_j\) = width of layer \(j\)
Example 4-11

Given the following permeability data from a core analysis report, calculate the average permeability of the reservoir.

<table>
<thead>
<tr>
<th>Depth, ft</th>
<th>Permeability, md</th>
</tr>
</thead>
<tbody>
<tr>
<td>3998-02</td>
<td>200</td>
</tr>
<tr>
<td>4002-04</td>
<td>130</td>
</tr>
<tr>
<td>4004-06</td>
<td>170</td>
</tr>
<tr>
<td>4006-08</td>
<td>180</td>
</tr>
<tr>
<td>4008-10</td>
<td>140</td>
</tr>
</tbody>
</table>

Solution

\[
\begin{align*}
\text{total } h_i k_i &= 2040 \\
\text{average } h_i k_i &= \frac{2040}{12} = 170 \text{ md}
\end{align*}
\]

Harmonic-Average Permeability

Permeability variations can occur laterally in a reservoir as well as in the vicinity of a well bore. Consider Figure 4-27 which shows an illustration of fluid flow through a series combination of beds with different permeabilities.

For a steady-state flow, the flow rate is constant and the total pressure drop \( \Delta p \) is equal to the sum of the pressure drops across each bed, or

\[
\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3
\]

Substituting for the pressure drop by applying Darcy’s equation, i.e., Equation 4-40, gives:
Canceling the identical terms and simplifying gives:

\[ k_{\text{avg}} = \frac{L}{(L/k)_1 + (L/k)_2 + (L/k)_3} \]

The above equation can be expressed in a more generalized form to give:

\[ k_{\text{avg}} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} (L/k)_i} \]  \hspace{1cm} (4-52)

where \( L_i \) = length of each bed

\( k_i \) = absolute permeability of each bed

In the radial system shown in Figure 4-28, the above averaging methodology can be applied to produce the following generalized expression:
The relationship in Equation 4-53 can be used as a basis for estimating a number of useful quantities in production work. For example, the effects of mud invasion, acidizing, or well shooting can be estimated from it.

**Example 4-12**

A hydrocarbon reservoir is characterized by five distinct formation segments that are connected in series. Each segment has the same formation thickness. The length and permeability of each section of the five-bed reservoir are given below:

\[
k_{\text{avg}} = \frac{\ln \left( \frac{r_e}{r_w} \right)}{\sum_{j=1}^{n} \frac{\ln \left( \frac{r_j}{r_{j-1}} \right)}{k_j}} \tag{4-53}
\]
Calculate the average permeability of the reservoir by assuming:

a. Linear flow system
b. Radial flow system

**Solution**

*For a linear system:*

<table>
<thead>
<tr>
<th>Length, ft</th>
<th>Permeability, md</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>300</td>
<td>30</td>
</tr>
<tr>
<td>500</td>
<td>20</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
</tbody>
</table>

\[
Li/k_i = 1350 \\
\Sigma L_i/k_i = 60.875
\]

Using Equation 4-52 gives:

\[
k_{avg} = \frac{1350}{60.875} = 22.18 \text{ md}
\]

*For a radial system:*

The solution of the radial system can be conveniently expressed in the following tabulated form. The solution is based on Equation 4-53 and assuming a wellbore radius of 0.25 ft:

<table>
<thead>
<tr>
<th>Segment</th>
<th>(r_i, \text{ ft})</th>
<th>(\ln(r_i/r_{B1}))</th>
<th>(k_i)</th>
<th>([\ln(r_i/r_{B1})]/k_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>well bore</td>
<td>0.25</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>6.397</td>
<td>80</td>
<td>0.080</td>
</tr>
<tr>
<td>2</td>
<td>350</td>
<td>0.847</td>
<td>50</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>650</td>
<td>0.619</td>
<td>30</td>
<td>0.021</td>
</tr>
<tr>
<td>4</td>
<td>1350</td>
<td>0.571</td>
<td>20</td>
<td>0.029</td>
</tr>
<tr>
<td>5</td>
<td>1350</td>
<td>0.160</td>
<td>10</td>
<td>0.016</td>
</tr>
</tbody>
</table>

\[0.163\]
From Equation 4-53,

\[ k_{\text{avg}} = \frac{\ln (1350/0.25)}{0.163} = 52.72 \text{ md} \]

**Geometric-Average Permeability**

Warren and Price (1961) illustrated experimentally that the most probable behavior of a heterogeneous formation approaches that of a uniform system having a permeability that is equal to the geometric average. The geometric average is defined mathematically by the following relationship:

\[
k_{\text{avg}} = \exp \left[ \frac{1}{n} \sum_{i=1}^{n} \left( h_i \ln (k_i) \right) \right] \tag{4 - 54}
\]

where \( k_i \) = permeability of core sample \( i \)

\( h_i \) = thickness of core sample \( i \)

\( n \) = total number of samples

If the thicknesses (\( h_i \)) of all core samples are the same, Equation 4-57 can be simplified as follows:

\[
k_{\text{avg}} = \left( \frac{1}{n} k_1 k_2 k_3 \ldots k_n \right)^{\frac{1}{n}} \tag{4 - 55}
\]

**Example 4-13**

Given the following core data, calculate the geometric average permeability:

<table>
<thead>
<tr>
<th>Sample</th>
<th>( h_i, \text{ ft} )</th>
<th>( k_i, \text{ md} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>70</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>50</td>
</tr>
<tr>
<td>9</td>
<td>1.5</td>
<td>35</td>
</tr>
<tr>
<td>10</td>
<td>0.5</td>
<td>20</td>
</tr>
</tbody>
</table>
### Solution

<table>
<thead>
<tr>
<th>Sample</th>
<th>( h_i ) ft</th>
<th>( k_i ) md</th>
<th>( h_i \cdot \ln(k_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>10</td>
<td>2.303</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>30</td>
<td>3.401</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>100</td>
<td>2.303</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>40</td>
<td>5.533</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>80</td>
<td>8.764</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>70</td>
<td>6.373</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>15</td>
<td>2.708</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>50</td>
<td>3.912</td>
</tr>
<tr>
<td>9</td>
<td>1.5</td>
<td>35</td>
<td>5.333</td>
</tr>
<tr>
<td>10</td>
<td>0.5</td>
<td>20</td>
<td>1.498</td>
</tr>
<tr>
<td>11.5</td>
<td></td>
<td></td>
<td>42.128</td>
</tr>
</tbody>
</table>

\[
 k_{\text{avg}} = \exp \left[ \frac{42.128}{11.5} \right] = 39 \text{ md} 
\]

### Absolute Permeability Correlations

The determination of connate water by capillary-pressure measurements has allowed the evaluation of connate-water values on samples of varying permeability and within a given reservoir to a wider extent and to a greater accuracy than was possible beforehand. These measurements have accumulated to the point where it is possible to correlate connate-water content with the permeability of the sample in a given reservoir and to a certain extent between reservoirs.

Calhoun (1976) suggested that in an ideal pore configuration of uniform structure, the irreducible connate water would be independent of permeability, lower permeabilities being obtained merely by a scaled reduction in particle size. In an actual porous system formed by deposition of graded particles or by some other natural means, the connate water might be expected to increase as permeability decreases. This conclusion results from the thought that lower permeabilities result from increasing non-uniformity of pore structure by a gradation of particles rather than by a scaled reduction of particles. In this sense, connate-water content is a function of permeability only insofar as permeability is dependent upon the variation of pore structure. Thus, for unconsolidated sands formed of uniform particles of one size, the connate-water content would be independent of permeability.
Calhoun (1976) pointed out that any correlation found between various reservoir properties would be anticipated to apply only within the rather narrow limits of a single reservoir or perhaps of a given formation. Beyond these bounds, a general correspondence between permeability and pore structure would not be known. It would be anticipated, however, that for formations of similar characteristics, a similar dependence of permeability on pore structure and, consequently, similar correlation of connate water and permeability would be found.

It has been generally considered for many years that connate water reached higher values in lower permeabilities. This observation amounted to nothing more than a trend. The data from capillary pressure measurements have indicated that the relationship is semi-logarithmic, although it is not yet certain from published data that this is the exact relationship. No generalizations are apparent from this amount of data, although it can now be quite generally stated that within a given reservoir the connate water (if an irreducible value) will increase proportionally to the decrease in the logarithm of the permeability. It is apparent, moreover, that one cannot state the value of connate water expected in any new formation unless one knows something of its pore makeup.

Experience indicates a general relationship between reservoir porosity ($\phi$) and irreducible water saturation ($S_{wc}$) provided the rock type and/or the grain size does not vary over the zone of interest. This relationship is defined by the equation

$$C = (S_{wc}) \phi$$

where $C$ is a constant for a particular rock type and/or grain size.

Several investigators suggest that the constant $C$ that describes the rock type can be correlated with the absolute permeability of the rock. Two commonly used empirical methods are the Timur equation and the Morris-Biggs equation.

**The Timur Equation**

Timur (1968) proposed the following expression for estimating the permeability from connate water saturation and porosity:

$$k = 8.58102 \frac{\phi^{4.4}}{S_{wc}^2}$$

(4 - 56)
The Morris-Biggs Equation

Morris and Biggs (1967) presented the following two expressions for estimating the permeability if oil and gas reservoirs:

For an oil reservoir:

$$k = 62.5 \left( \frac{\phi^3}{S_{wc}} \right)^2$$  \hspace{1cm} (4 - 57)

For a gas reservoir:

$$k = 2.5 \left( \frac{\phi^3}{S_{wc}} \right)^2$$  \hspace{1cm} (4 - 58)

where $k$ = absolute permeability, Darcy  
$\phi$ = porosity, fraction 
$S_{wc}$ = connate-water saturation, fraction

Example 4-14

Estimate the absolute permeability of an oil zone with a connate-water saturation and average porosity of 25% and 19%, respectively.

Solution

Applying the Timur equation:

$$k = 8.58102 \frac{(0.19)^{4.4}}{(0.25)^2} = 0.0921 \text{ Darcy}$$

From the Morris and Biggs correlation:

$$k = 62.5 \left( \frac{(0.29)^3}{0.25} \right)^2 = 0.047 \text{ Darcy}$$

In the previous discussion of Darcy’s Law and absolute permeability measurements, it was assumed that the entire porous medium is fully saturated with a single phase, i.e., 100% saturation. In hydrocarbon reservoir, however, the rocks are usually saturated with two or more fluids. Therefore, the concept of absolute permeability must be modified to describe the fluid flowing behavior when more than one fluid is present.
in the reservoir. If a core sample is partially saturated with a fluid (other than the test fluid) and both saturations are maintained constant throughout the flow, the measured permeability to the test fluid will be reduced below the permeability which could be measured if the core were 100 percent saturated with the test fluid.

As the saturation of a particular phase decreases, the permeability to that phase also decreases. The measured permeability is referred to as the effective permeability and is a relative measure of the conductance of the porous medium for one fluid when the medium is saturated with more than one fluid. This implies that the effective permeability is an associated property with each reservoir fluid, i.e., gas, oil, and water. These effective permeabilities for the three reservoir fluids are represented by:

\[ k_g = \text{effective gas permeability} \]
\[ k_o = \text{effective oil permeability} \]
\[ k_w = \text{effective water permeability} \]

One of the phenomena of multiphase effective permeabilities is that the sum of the effective permeabilities is always less than or equal to the absolute permeability, i.e.,

\[ k_g + k_o + k_w \leq k \]

The effective permeability is used mathematically in Darcy's Law in place of the absolute permeability. For example, the expression for flow through the linear system under a partial saturation of oil is written

\[ q_o = \frac{k_o}{\mu_o} \frac{A \left( p_1 - p_2 \right)}{L} \]  

(4 - 59)

where \( q_o \) = oil flow rate, cc/sec
\( \mu_o \) = oil viscosity, cm
\( k_o \) = oil effective permeability, Darcys

Effective permeabilities are normally measured directly in the laboratory on small core samples. Owing to the many possible combinations of saturation for a single medium, however, laboratory data are usually summarized and reported as relative permeability. Relative permeability is defined as the ratio of the effective permeability to a given fluid at a definite saturation to the permeability at 100% saturation. The terminolo-
The most widely used is simply $k_g/k$, $k_o/k$, $k_w/k$, meaning the relative permeability to gas, oil, and water, respectively. Since $k$ is a constant for a given porous material, the relative permeability varies with the fluid saturation in the same fashion as does the effective permeability. The relative permeability to a fluid will vary from a value of zero at some low saturation of that fluid to a value of 1.0 at 100% saturation of that fluid. Thus, the relative permeability can be expressed symbolically as

$$
\begin{align*}
  k_{rg} &= \frac{k_g}{k} \\
  k_{ro} &= \frac{k_o}{k} \\
  k_{rw} &= \frac{k_w}{k}
\end{align*}
$$

which are relative permeabilities to gas, oil, and water, respectively. A comprehensive treatment of the relative permeability is presented in Chapter 5.

**ROCK COMpressibility**

A reservoir thousands of feet underground is subjected to an overburden pressure caused by the weight of the overlying formations. Overburden pressures vary from area to area depending on factors such as depth, nature of the structure, consolidation of the formation, and possibly the geologic age and history of the rocks. Depth of the formation is the most important consideration, and a typical value of overburden pressure is approximately one psi per foot of depth.

The weight of the overburden simply applies a compressive force to the reservoir. The pressure in the rock pore spaces does not normally approach the overburden pressure. A typical pore pressure, commonly referred to as the reservoir pressure, is approximately 0.5 psi per foot of depth, assuming that the reservoir is sufficiently consolidated so the overburden pressure is not transmitted to the fluids in the pore spaces.

The pressure difference between overburden and internal pore pressure is referred to as the effective overburden pressure. During pressure depletion operations, the internal pore pressure decreases and, therefore, the effective overburden pressure increases. This increase causes the following effects:

- The bulk volume of the reservoir rock is reduced.
- Sand grains within the pore spaces expand.
These two volume changes tend to reduce the pore space and, therefore, the porosity of the rock. Often these data exhibit relationships with both porosity and the effective overburden pressure. Compressibility typically decreases with increasing porosity and effective overburden pressure.

Geertsma (1957) points out that there are three different types of compressibility that must be distinguished in rocks:

- **Rock-matrix compressibility,** $c_r$
  
  Is defined as the fractional change in volume of the solid rock material (grains) with a unit change in pressure. Mathematically, the rock compressibility coefficient is given by

  \[ c_r = -\frac{1}{V_r} \left( \frac{\partial V_r}{\partial p} \right)_T \]  

  where $c_r = \text{rock-matrix compressibility, psi}^{-1}$
  
  $V_r = \text{volume of solids}$

  The subscript T indicates that the derivative is taken at constant temperature.

- **Rock-bulk compressibility,** $c_B$
  
  Is defined as the fractional change in volume of the bulk volume of the rock with a unit change in pressure. The rock-bulk compressibility is defined mathematically by:

  \[ c_B = -\frac{1}{V_B} \left( \frac{\partial V_B}{\partial p} \right)_T \]  

  where $c_B = \text{rock-bulk compressibility coefficient, psi}^{-1}$
  
  $V_B = \text{bulk volume}$

- **Pore compressibility,** $c_p$
  
  The pore compressibility coefficient is defined as the fractional change in pore volume of the rock with a unit change in pressure and given by the following relationship:

  \[ c_p = -\frac{1}{V_p} \left( \frac{\partial V_p}{\partial p} \right)_T \]  

  \[ (4 - 62) \]
where \( p \) = pore pressure, psi
\( c_p \) = pore compressibility coefficient, psi\(^{-1}\)
\( V_p \) = pore volume

Equation 4-62 can be expressed in terms of the porosity \( \phi \) by noting that \( \phi \) increases with the increase in the pore pressure; or:

\[
c_p = \frac{1}{\phi} \frac{\partial \phi}{\partial p}
\]

For most petroleum reservoirs, the rock and bulk compressibility are considered small in comparison with the pore compressibility \( c_p \). The *formation compressibility* \( c_f \) is the term commonly used to describe the total compressibility of the formation and is set equal to \( c_p \), i.e.:

\[
c_f = c_p = \frac{1}{\phi} \frac{\partial \phi}{\partial p}
\]  \hspace{1cm} (4-63)

Typical values for the formation compressibility range from \( 3 \times 10^{-6} \) to \( 25 \times 10^{-6} \) psi\(^{-1}\). Equation 4-62 can be rewritten as:

\[
c_f = \frac{1}{V_p} \frac{\Delta V_p}{\Delta p}
\]

or

\[
\Delta V_p = c_f V_p \Delta p
\]  \hspace{1cm} (4-64)

where \( \Delta V_p \) and \( \Delta p \) are the change in the pore volume and pore pressure, respectively.

Geertsma (1957) suggested that the bulk compressibility \( c_B \) is related to the pore compressibility \( c_p \) by the following expression.

\[
c_B \equiv c_p \phi
\]  \hspace{1cm} (4-65)

Geertsma has stated that in a reservoir only the vertical component of hydraulic stress is constant and that the stress components in the horizontal plane are characterized by the boundary condition that there is no bulk deformation in those directions. For those boundary conditions, he developed the following approximation for sandstones:

\[
 c_p \text{ (reservoir)} = \frac{1}{2} c_p \text{ (laboratory)}
\]
Example 4-15

Calculate the reduction in the pore volume of a reservoir due to a pressure drop of 10 psi. The reservoir original pore volume is one million barrels with an estimated formation compressibility of $10 \times 10^{-6} \text{ psi}^{-1}$.

Solution

Applying Equation 4-64 gives

$$\Delta V_p = (10 \times 10^{-6})(1 \times 10^6)(10) = 100 \text{ bbl}$$

Although the above value is small, it becomes an important factor in undersaturated reservoirs when calculations are made to determine initial oil-in-place and aquifer contents.

The reduction in the pore volume due to pressure decline can also be expressed in terms of the changes in the reservoir porosity. Equation 4-63 can be rearranged, to give:

$$c_f \partial p = \left( \frac{1}{\phi} \right) \partial \phi$$

Integrating the above relation gives:

$$c_f \int_{p_o}^{p} \partial p = \int_{\phi_o}^{\phi} \frac{\partial \phi}{\phi}$$

$$c_f (p - p_o) = \ln \left( \frac{\phi}{\phi_o} \right)$$

or:

$$\phi = \phi_o e^{c_f (p - p_o)} \quad (4-66)$$

where $p_o =$ original pressure, psi
$\phi_o =$ original porosity
$p =$ current pressure, psi
$\phi =$ porosity at pressure $p$
Noting that the $e^x$ expansion series is expressed as:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Using the expansion series and truncating the series after the first two terms, gives:

$$\phi = \phi_o [1 + c_f (p - p_o)] \quad (4-67)$$

**Example 4-16**

Given the following data:

- $c_f = 10 \times 10^{-6}$
- original pressure = 5000 psi
- original porosity = 18%
- current pressure = 4500 psi

Calculate the porosity at 4500 psi.

**Solution**

$$\phi = 0.18 [1 + (10 \times 10^{-6})(4500 - 5000)] = 0.179$$

It should be pointed out that the total reservoir compressibility $c_t$ is extensively used in the transient flow equation and the material balance equation as defined by the following expression:

$$c_t = S_o c_o + S_w c_w + S_g c_g + c_f \quad (4-68)$$

where $S_o, S_w, S_g$ = oil, water, and gas saturation

- $c_o$ = oil compressibility, psi$^{-1}$
- $c_w$ = water compressibility, psi$^{-1}$
- $c_g$ = gas compressibility, psi$^{-1}$
- $c_t$ = total reservoir compressibility

For undersaturated oil reservoirs, the reservoir pressure is above the bubble-point pressure, i.e., no initial gas cap, which reduces Equation 4-68 to:

$$c_t = S_o c_o + S_w c_w + c_f$$
In general, the formation compressibility $c_f$ is the same order of magnitude as the compressibility of the oil and water and, therefore, cannot be regulated.

Several authors have attempted to correlate the pore compressibility with various parameters including the formation porosity. Hall (1953) correlated the pore compressibility with porosity as given by the following relationship:

$$c_f = \frac{1.782}{\phi^{0.438}} \times 10^{-6}$$

(4-69)

where $c_f =$ formation compressibility, psi$^{-1}$

$\phi =$ porosity, fraction

Newman (1973) used 79 samples for consolidated sandstones and limestones to develop a correlation between the formation compressibility and porosity. The proposed generalized hyperbolic form of the equation is:

$$c_f = \frac{a}{[1 + cb\phi]}$$

where

**For consolidated sandstones**

- $a = 97.32 \times 10^{-6}$
- $b = 0.699993$
- $c = 79.8181$

**For limestones**

- $a = 0.8535$
- $b = 1.075$
- $c = 2.202 \times 10^6$

**Example 4-17**

Estimate the compressibility coefficient of a sandstone formation that is characterized by a porosity of 0.2, using:

a. Hall’s correlation

b. Newman’s correlation
Solution

a. Hall’s correlations:

$$c_f = (1.782/0.2^{0.438}) \times 10^{-6} = 3.606 \times 10^{-6} \text{ psi}^{-1}$$

b. Newman’s correlation:

$$c_f = \frac{97.32 \times 10^{-6}}{[1 + (0.699993)(79.8181)(0.2)]^{1/0.699993}} = 2.74 \times 10^{-6} \text{ psi}^{-1}$$

**NET PAY THICKNESS**

A fundamental prerequisite to reservoir performance prediction is a satisfactory knowledge of the volume of oil originally in place. The reservoir is necessarily confined to certain geologic and fluid boundaries, i.e., GOC, WOC, and GWC, so accuracy is imperative. Within the confines of such boundaries, oil is contained in what is commonly referred to as *Gross Pay*. Net Pay is that part of the reservoir thickness which contributes to oil recovery and is defined by imposing the following criteria:

- Lower limit of porosity
- Lower limit of permeability
- Upper limit of water saturation

All available measurements performed on reservoir samples and in wells, such as core analysis and well logs, are extensively used in evaluating the reservoir net thickness.

The choice of lower limits of porosity and permeability will depend upon such individual characteristics as

- Total reservoir volume
- Total range of permeability values
- Total range of porosity values
- Distribution of the permeability and porosity values
It has been proposed that most reservoirs are laid down in a body of water by a long-term process, spanning a variety of depositional environments, in both time and space. As a result of subsequent physical and chemical reorganization, such as compaction, solution, dolomitization and cementation, the reservoir characteristics are further changed. Thus the heterogeneity of reservoirs is, for the most part, dependent upon the depositional environments and subsequent events.

The main geologic characteristic of all the physical rock properties that have a bearing on reservoir behavior when producing oil and gas is the extreme variability in such properties within the reservoir itself, both laterally and vertically, and within short distances. It is important to recognize that there are no homogeneous reservoirs, only varying degrees of heterogeneity.

The reservoir heterogeneity is then defined as a variation in reservoir properties as a function of space. Ideally, if the reservoir is homogeneous, measuring a reservoir property at any location will allow us to fully describe the reservoir. The task of reservoir description is very simple for homogeneous reservoirs. On the other hand, if the reservoir is heterogeneous, the reservoir properties vary as a function of a spatial location. These properties may include permeability, porosity, thickness, saturation, faults and fractures, rock facies and rock characteristics. For a proper reservoir description, we need to predict the variation in these reservoir properties as a function of spatial locations. There are essentially two types of heterogeneity:

- Vertical heterogeneity
- Areal heterogeneity

Geostatistical methods are used extensively in the petroleum industry to quantitatively describe the two types of the reservoir heterogeneity. It is obvious that the reservoir may be nonuniform in all intensive properties such as permeability, porosity, wettability, and connate water saturation. We will discuss heterogeneity of the reservoir in terms of permeability.
Vertical Heterogeneity

One of the first problems encountered by the reservoir engineer in predicting or interpreting fluid displacement behavior during secondary recovery and enhanced oil recovery processes is that of organizing and using the large amount of data available from core analysis. Permeabilities pose particular problems in organization because they usually vary by more than an order of magnitude between different strata. The engineer must be able then to:

- Describe the degree of the vertical heterogeneity in mathematical terms, and
- Describe and define the proper permeability stratification of the pay zone. This task is commonly called the zoning or layering problem.

It is appropriate to be able to describe the degree of heterogeneity within a particular system in quantitative terms. The degree of homogeneity of a reservoir property is a number that characterizes the departure from uniformity or constancy of that particular measured property through the thickness of reservoir. A formation is said to have a uniformity coefficient of zero in a specified property when that property is constant throughout the formation thickness. A completely heterogeneous formation has a uniformity coefficient of unity. Between the two extremes, formations have uniformity coefficients comprised between zero and one. The following are the two most widely used descriptors of the vertical heterogeneity of the formation:

- Dykstra-Parsons permeability variation $V$
- Lorenz coefficient $L$

The Dykstra-Parsons Permeability Variation

Dykstra and Parsons (1950) introduced the concept of the permeability variation coefficient $V$ which is a statistical measure of non-uniformity of a set of data. It is generally applied to the property of permeability but can be extended to treat other rock properties. It is generally recognized that the permeability data are log-normally distributed. That is, the geologic processes that create permeability in reservoir rocks appear to leave permeabilities distributed around the geometric mean. Dykstra and Parsons recognized this feature and introduced the permeability variation that characterizes a particular distribution. The required computational steps for determining the coefficient $V$ are summarized below:
Solution

Step 1. Calculate residual liquid saturation $S_{lc}$.

$$S_{lc} = S_{wc} + S_{org} = 0.25 + 0.23 = 0.48$$

Step 2. Generate relative permeability and capillary pressure data for oil-water system by applying Equations 5-22 through 5-24.

<table>
<thead>
<tr>
<th>$S_w$</th>
<th>$k_{ro}$ Equations 5-22</th>
<th>$k_{rw}$ Equation 5-23</th>
<th>$p_c$ Equation 5-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.850</td>
<td>0.000</td>
<td>20.00</td>
</tr>
<tr>
<td>0.30</td>
<td>0.754</td>
<td>0.018</td>
<td>18.19</td>
</tr>
<tr>
<td>0.40</td>
<td>0.557</td>
<td>0.092</td>
<td>14.33</td>
</tr>
<tr>
<td>0.50</td>
<td>0.352</td>
<td>0.198</td>
<td>9.97</td>
</tr>
<tr>
<td>0.60</td>
<td>0.131</td>
<td>0.327</td>
<td>4.57</td>
</tr>
<tr>
<td>0.65</td>
<td>0.000</td>
<td>0.400</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Step 3. Apply Equations 5-25 through 5-27 to determine the relative permeability and capillary data for the gas-oil system.

<table>
<thead>
<tr>
<th>$S_g$</th>
<th>$k_{ro}$ Equation 5-25</th>
<th>$k_{rg}$ Equation 5-26</th>
<th>$p_c$ Equation 5-27</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.600</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.10</td>
<td>0.524</td>
<td>0.248</td>
<td>9.56</td>
</tr>
<tr>
<td>0.20</td>
<td>0.378</td>
<td>0.479</td>
<td>16.76</td>
</tr>
<tr>
<td>0.30</td>
<td>0.241</td>
<td>0.650</td>
<td>21.74</td>
</tr>
<tr>
<td>0.40</td>
<td>0.117</td>
<td>0.796</td>
<td>25.81</td>
</tr>
<tr>
<td>0.52</td>
<td>0.000</td>
<td>0.95</td>
<td>30.00</td>
</tr>
</tbody>
</table>

**RELATIVE PERMEABILITY RATIO**

Another useful relationship that derives from the relative permeability concept is the relative (or effective) permeability ratio. This quantity lends itself more readily to analysis and to the correlation of flow performances than does relative permeability itself. The relative permeability ratio expresses the ability of a reservoir to permit flow of one fluid as related to its ability to permit flow of another fluid under the same circumstances. The two most useful permeability ratios are $k_{rg}/k_{ro}$, the relative permeability-
ty to gas with respect to that to oil and $k_{rg}/k_{ro}$ the relative permeability to water with respect to that to oil, it being understood that both quantities in the ratio are determined simultaneously on a given system. The relative permeability ratio may vary in magnitude from zero to infinity.

In describing two-phase flow mathematically, it is always the relative permeability ratio (e.g., $k_{rg}/k_{ro}$ or $k_{ro}/k_{rw}$) that is used in the flow equations. Because the wide range of the relative permeability ratio values, the permeability ratio is usually plotted on the log scale of semilog paper as a function of the saturation. Like many relative permeability ratio curves, the central or the main portion of the curve is quite linear.

Figure 5-6 shows a plot of $k_{rg}/k_{ro}$ versus gas saturation. It has become common usage to express the central straight-line portion of the relationship in the following analytical form:

$$\frac{k_{rg}}{k_{ro}} = a e^{bS_g}$$

(5 - 28)

The constants $a$ and $b$ may be determined by selecting the coordinate of two different points on the straight-line portion of the curve and substituting in Equation 5-28. The resulting two equations can be solved simultaneously for the constants $a$ and $b$. To find the coefficients of Equation 5-28 for the straight-line portion of Figure 5-6, select the following two points:

**Point 1:** at $S_g = 0.2$, the relative permeability ratio $k_{rg}/k_{ro} = 0.07$

**Point 2:** at $S_g = 0.4$, the relative permeability ratio $k_{rg}/k_{ro} = 0.70$

Imposing the above points on Equation 5-28, gives:

$$0.07 = a e^{0.2b}$$

$$0.70 = a e^{0.4b}$$

Solving simultaneously gives:

- The intercept $a = 0.0070$
- The slope $b = 11.513$
Figure 5-6. $k_{rg}/k_o$ as a function of saturation.
or

\[
\frac{k_{rg}}{k_{ro}} = 0.0070 \ e^{11.513S_g}
\]

In a similar manner, Figure 5-7 shows a semilog plot of \(k_{ro}/k_{rw}\) versus water saturation.

The middle straight-line portion of the curve is expressed by a relationship similar to that of Equation 5-28

\[
\frac{k_{ro}}{k_{rw}} = a \ e^{bS_w} \quad (5-29)
\]

where the slope \(b\) has a negative value.

**DYNAMIC PSEUDO-RELATIVE PERMEABILITIES**

For a multilayered reservoir with each layer as described by a set of relative permeability curves, it is possible to treat the reservoir by a single layer...
that is characterized by a weighted-average porosity, absolute permeability, and a set of dynamic pseudo-relative permeability curves. These averaging properties are calculated by applying the following set of relationships:

**Average Porosity**

\[
\phi_{avg} = \frac{\sum_{i=1}^{N} \phi_i h_i}{\sum_{i=1}^{N} h_i}
\]  

(5 - 30)

**Average Absolute Permeability**

\[
k_{avg} = \frac{\sum_{i=1}^{N} k_i h_i}{\sum_{i=1}^{N} h_i}
\]  

(5 - 31)

**Average Relative Permeability for the Wetting Phase**

\[
\bar{k}_{rw} = \frac{\sum_{i=1}^{N} (k h)_i (k_{rw})_i}{\sum_{i=1}^{N} (k h)_i}
\]  

(5 - 32)

**Average Relative Permeability for the Nonwetting Phase**

\[
\bar{k}_{r nw} = \frac{\sum_{i=1}^{N} (k h)_i (k_{r nw})_i}{\sum_{i=1}^{N} (k h)_i}
\]  

(5 - 33)

The corresponding average saturations should be determined by using Equations 4-16 through 4-18. These equations are given below for convenience:
Average Oil Saturation

\[
\bar{S}_o = \frac{\sum_{i=1}^{n} \phi_i h_i S_{oi}}{\sum_{i=1}^{n} \phi_i h_i}
\]

Average Water Saturation

\[
\bar{S}_w = \frac{\sum_{i=1}^{n} \phi_i h_i S_{wi}}{\sum_{i=1}^{n} \phi_i h_i}
\]

Average Gas Saturation

\[
\bar{S}_g = \frac{\sum_{i=1}^{n} \phi_i h_i S_{gi}}{\sum_{i=1}^{n} \phi_i h_i}
\]

where  
- \( n \) = total number of layers  
- \( h_i \) = thickness of layer \( i \)  
- \( k_i \) = absolute permeability of layer \( i \)  
- \( k_{rw} \) = average relative permeability of the wetting phase  
- \( k_{rnw} \) = average relative permeability of the nonwetting phase

In Equations 5-22 and 5-23, the subscripts \( w \) and \( n_w \) represent wetting and nonwetting, respectively. The resulting dynamic pseudo-relative permeability curves are then used in a single-layer model. The objective of the single-layer model is to produce results similar to those from the multilayered, cross-sectional model.
NORMALIZATION AND AVERAGING
RELATIVE PERMEABILITY DATA

Results of relative permeability tests performed on several core samples of a reservoir rock often vary. Therefore, it is necessary to average the relative permeability data obtained on individual rock samples. Prior to usage for oil recovery prediction, the relative permeability curves should first be normalized to remove the effect of different initial water and critical oil saturations. The relative permeability can then be de-normalized and assigned to different regions of the reservoir based on the existing critical fluid saturation for each reservoir region.

The most generally used method adjusts all data to reflect assigned end values, determines an average adjusted curve and finally constructs an average curve to reflect reservoir conditions. These procedures are commonly described as normalizing and de-normalizing the relative permeability data.

To perform the normalization procedure, it is helpful to set up the calculation steps for each core sample i in a tabulated form as shown below:

\[
\begin{array}{cccccc}
(1) & (2) & (3) & (4) & (5) & (6) \\
S_w & k_{ro} & k_{rw} & S_w^* = \frac{S_w - S_{wc}}{1-S_{wc}-S_{oc}} & k_{ro}^* = \frac{k_{ro}}{(k_{ro})_{S_{wc}}} & k_{rw}^* = \frac{k_{rw}}{(k_{rw})_{S_{oc}}}
\end{array}
\]

The following normalization methodology describes the necessary steps for a water-oil system as outlined in the above table.

Step 1. Select several values of \(S_w\) starting at \(S_{wc}\) (column 1), and list the corresponding values of \(k_{ro}\) and \(k_{rw}\) in columns 2 and 3.

Step 2. Calculate the normalized water saturation \(S_{w}^*\) for each set of relative permeability curves and list the calculated values in column 4 by using the following expression:

\[
S_{w}^* = \frac{S_w - S_{wc}}{1-S_{wc}-S_{oc}}
\]
where \( S_{oc} \) = critical oil saturation
\( S_{wc} \) = connate water saturation
\( S_{w}^{*} \) = normalized water saturation

**Step 3.** Calculate the normalized relative permeability for the oil phase at different water saturation by using the relation (column 5):

\[
k_{ro}^{*} = \frac{k_{ro}}{(k_{ro})_{Swc}}
\]  \( (5 - 35) \)

where \( k_{ro} \) = relative permeability of oil at different \( S_{w} \)
\( (k_{ro})_{Swc} \) = relative permeability of oil at connate water saturation
\( k_{ro}^{*} \) = normalized relative permeability of oil

**Step 4.** Normalize the relative permeability of the water phase by applying the following expression and document results of the calculation in column 6

\[
k_{rw}^{*} = \frac{k_{rw}}{(k_{rw})_{Soc}}
\]  \( (5 - 36) \)

where \( (k_{rw})_{Soc} \) is the relative permeability of water at the critical oil saturation.

**Step 5.** Using regular Cartesian coordinate, plot the normalized \( k_{ro}^{*} \) and \( k_{rw}^{*} \) versus \( S_{w}^{*} \) for all core samples on the same graph.

**Step 6.** Determine the average normalized relative permeability values for oil and water as a function of the normalized water saturation by select arbitrary values of \( S_{w}^{*} \) and calculate the average of \( k_{ro}^{*} \) and \( k_{rw}^{*} \) by applying the following relationships:

\[
(k_{ro})_{avg}^{*} = \frac{\sum_{i=1}^{n} (h k_{ro}^{*})_{i}}{\sum_{i=1}^{n} (h k)_{i}}
\]  \( (5 - 37) \)
Step 7. The last step in this methodology involves de-normalizing the average curve to reflect actual reservoir and conditions of $S_{wc}$ and $S_{oc}$. These parameters are the most critical part of the methodology and, therefore, a major effort should be spent in determining representative values. The $S_{wc}$ and $S_{oc}$ are usually determined by averaging the core data, log analysis, or correlations, versus graphs, such as: $(k_{ro})_{S_{wc}}$ vs. $S_{wc}$, $(k_{ro})_{S_{oc}}$ vs. $S_{oc}$, and $S_{oc}$ vs. $S_{wc}$ which should be constructed to determine if a significant correlation exists. Often, plots of $S_{wc}$ and $S_{oc}$ versus log $Z/k/f$ may demonstrate a reliable correlation to determine end-point saturations as shown schematically in Figure 5-8. When representative end values have been estimated, it is again convenient to perform the de-normalization calculations in a tabular form as illustrated below:

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{wc}$</td>
<td>$(k_{ro})_{avg}$</td>
<td>$(k_{ro})_{avg}$</td>
<td>$S_{wc} = S_{wc}^* (1 - S_{wc} - S_{oc}) + S_{wc}$</td>
<td>$k_{ro} = (k_{ro})_{avg}$</td>
<td>$(k_{ro})_{avg}$</td>
<td>$k_{ro} = (k_{ro})_{avg}$</td>
</tr>
<tr>
<td>$S_{oc}$</td>
<td>$(k_{ro})<em>{S</em>{wc}}$</td>
<td>$(k_{ro})<em>{S</em>{oc}}$</td>
<td>$S_{oc} = S_{wc}^* (1 - S_{wc} - S_{oc}) + S_{oc}$</td>
<td>$k_{ro} = (k_{ro})_{avg}$</td>
<td>$(k_{ro})_{avg}$</td>
<td>$k_{ro} = (k_{ro})_{avg}$</td>
</tr>
</tbody>
</table>

Where $(k_{ro})_{S_{wc}}$ and $(k_{ro})_{S_{oc}}$ are the average relative permeability of oil and water at connate water and critical oil, respectively, and given by:

\[
(k_{ro})_{S_{wc}} = \frac{\sum_{i=1}^{n} (h k (k_{ro})_{S_{wc}})}{\sum_{i=1}^{n} (h k)_i} \quad (5 - 38)
\]

\[
(k_{ro})_{S_{oc}} = \frac{\sum_{i=1}^{n} (h k (k_{ro})_{S_{oc}})}{\sum_{i=1}^{n} (h k)_i} \quad (5 - 39)
\]
Example 5-6

Relative permeability measurements are made on three core samples. The measured data are summarized below:

<table>
<thead>
<tr>
<th></th>
<th>Core Sample #1</th>
<th>Core Sample #2</th>
<th>Core Sample #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>1 ft</td>
<td>1 ft</td>
<td>1 ft</td>
</tr>
<tr>
<td>( k )</td>
<td>100 md</td>
<td>80 md</td>
<td>150 md</td>
</tr>
<tr>
<td>( S_{oc} )</td>
<td>0.35</td>
<td>0.28</td>
<td>0.35</td>
</tr>
<tr>
<td>( S_{wc} )</td>
<td>0.25</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td>( k_{ro} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{rw} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( S_w )</td>
<td>0.20</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>( k_{rw} )</td>
<td>—</td>
<td>0.850*</td>
<td>0.20</td>
</tr>
<tr>
<td>( k_{ro} )</td>
<td>0.754</td>
<td>0.352</td>
<td>0.35</td>
</tr>
<tr>
<td>( k_{rw} )</td>
<td>0.018</td>
<td>0.198</td>
<td>0.25</td>
</tr>
<tr>
<td>( k_{ro} )</td>
<td>0.800</td>
<td>0.393</td>
<td>0.394</td>
</tr>
<tr>
<td>( k_{rw} )</td>
<td>0.000</td>
<td>0.393</td>
<td>0.394</td>
</tr>
<tr>
<td>( k_{rw} )</td>
<td>0.000</td>
<td>0.500*</td>
<td></td>
</tr>
</tbody>
</table>

*Values at critical saturations

It is believed that a connate water saturation of 0.27 and a critical oil saturation of 30% better describe the formation. Generate the oil and water relative permeability data using the new critical saturations.
Solution

Step 1. Calculate the normalized water saturation for each core sample by using Equation 5-36.

<table>
<thead>
<tr>
<th>$S_w^*$</th>
<th>Core Sample #1</th>
<th>Core Sample #2</th>
<th>Core Sample #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>—</td>
<td>—</td>
<td>0.000</td>
</tr>
<tr>
<td>0.25</td>
<td>0.000</td>
<td>—</td>
<td>0.111</td>
</tr>
<tr>
<td>0.30</td>
<td>0.125</td>
<td>0.000</td>
<td>0.222</td>
</tr>
<tr>
<td>0.40</td>
<td>0.375</td>
<td>0.238</td>
<td>0.444</td>
</tr>
<tr>
<td>0.50</td>
<td>0.625</td>
<td>0.476</td>
<td>0.667</td>
</tr>
<tr>
<td>0.60</td>
<td>0.875</td>
<td>0.714</td>
<td>0.889</td>
</tr>
<tr>
<td>0.65</td>
<td>1.000</td>
<td>0.833</td>
<td>1.000</td>
</tr>
<tr>
<td>0.72</td>
<td>—</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Step 2. Determine relative permeability values at critical saturation for each core sample.

<table>
<thead>
<tr>
<th></th>
<th>Core 1</th>
<th>Core 2</th>
<th>Core 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(k_{ro})_{Swc}$</td>
<td>0.850</td>
<td>0.800</td>
<td>1.000</td>
</tr>
<tr>
<td>$(k_{rw})_{Sor}$</td>
<td>0.400</td>
<td>0.500</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Step 3. Calculate $(\bar{k}_{ro})_{Swc}$ and $(\bar{k}_{rw})_{Sor}$ by applying Equations 5-39 and 5-40 to give:
- $(\bar{k}_{ro})_{Swc} = 0.906$
- $(\bar{k}_{rw})_{Sor} = 0.402$

Step 4. Calculate the normalized $k^*_{ro}$ and $k^*_{rw}$ for all core samples:

<table>
<thead>
<tr>
<th>$S_w$</th>
<th>$S_w^*$</th>
<th>Core 1 $k^*_{ro}$</th>
<th>Core 1 $k^*_{rw}$</th>
<th>Core 2 $k^*_{ro}$</th>
<th>Core 2 $k^*_{rw}$</th>
<th>Core 3 $k^*_{ro}$</th>
<th>Core 3 $k^*_{rw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.25</td>
<td>0.000</td>
<td>1.000</td>
<td>0</td>
<td>—</td>
<td>—</td>
<td>0.111</td>
<td>0.872</td>
</tr>
<tr>
<td>0.30</td>
<td>0.125</td>
<td>0.887</td>
<td>0.045</td>
<td>0.000</td>
<td>1.000</td>
<td>0</td>
<td>0.222</td>
</tr>
<tr>
<td>0.40</td>
<td>0.375</td>
<td>0.655</td>
<td>0.230</td>
<td>0.238</td>
<td>0.741</td>
<td>0.154</td>
<td>0.444</td>
</tr>
<tr>
<td>0.50</td>
<td>0.625</td>
<td>0.414</td>
<td>0.495</td>
<td>0.476</td>
<td>0.491</td>
<td>0.382</td>
<td>0.667</td>
</tr>
<tr>
<td>0.60</td>
<td>0.875</td>
<td>0.154</td>
<td>0.818</td>
<td>0.714</td>
<td>0.252</td>
<td>0.646</td>
<td>0.889</td>
</tr>
<tr>
<td>0.65</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.833</td>
<td>0.139</td>
<td>0.788</td>
<td>1.000</td>
</tr>
<tr>
<td>0.72</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>—</td>
</tr>
</tbody>
</table>
Step 5. Plot the normalized values of $k_{ro}^*$ and $k_{rw}^*$ versus $S_w^*$ for each core on a regular graph paper as shown in Figure 5-9.

Step 6. Select arbitrary values of $S_w^*$ and calculate the average $k_{ro}^*$ and $k_{rw}^*$ by applying Equations 5-37 and 5-38.

<table>
<thead>
<tr>
<th>$S_w^*$</th>
<th>$k_{ro}^*$</th>
<th>$\langle k_{ro}^*\rangle_{\text{Avg}}$</th>
<th>$k_{rw}^*$</th>
<th>$\langle k_{rw}^*\rangle_{\text{Avg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Core 1</td>
<td>Core 2</td>
<td>Core 3</td>
<td>Core 1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.91</td>
<td>0.88</td>
<td>0.93</td>
<td>0.912</td>
</tr>
<tr>
<td>0.2</td>
<td>0.81</td>
<td>0.78</td>
<td>0.85</td>
<td>0.821</td>
</tr>
<tr>
<td>0.3</td>
<td>0.72</td>
<td>0.67</td>
<td>0.78</td>
<td>0.735</td>
</tr>
<tr>
<td>0.4</td>
<td>0.63</td>
<td>0.51</td>
<td>0.70</td>
<td>0.633</td>
</tr>
<tr>
<td>0.5</td>
<td>0.54</td>
<td>0.46</td>
<td>0.61</td>
<td>0.552</td>
</tr>
<tr>
<td>0.6</td>
<td>0.44</td>
<td>0.37</td>
<td>0.52</td>
<td>0.459</td>
</tr>
<tr>
<td>0.7</td>
<td>0.33</td>
<td>0.27</td>
<td>0.42</td>
<td>0.356</td>
</tr>
<tr>
<td>0.8</td>
<td>0.23</td>
<td>0.17</td>
<td>0.32</td>
<td>0.256</td>
</tr>
<tr>
<td>0.9</td>
<td>0.12</td>
<td>0.07</td>
<td>0.18</td>
<td>0.135</td>
</tr>
</tbody>
</table>

![Figure 5-9](image.png) Averaging relative permeability data.
Step 7. Using the desired formation $S_{oc}$ and $S_{wc}$ (i.e., $S_{oc} = 0.30$, $S_{wc} = 0.27$), de-normalize the data to generate the required relative permeability data as shown below:

\[
S^*_w = \frac{k^*_r}{k^*_ro} = \frac{S^*_w}{S^*_w (1 - S_{wc} - S_{oc}) + S_{wc}}
\]

<table>
<thead>
<tr>
<th>$S^*_w$</th>
<th>$(k^*_r/ro)_avg$</th>
<th>$(k^*_rw)_avg$</th>
<th>$\frac{S^*<em>w}{(1 - S</em>{wc} - S_{oc}) + S_{wc}}$</th>
<th>$k_{ro} = 0.906$</th>
<th>$k_{rw} = 0.402$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.912</td>
<td>0.038</td>
<td>0.313</td>
<td>0.826</td>
<td>0.015</td>
</tr>
<tr>
<td>0.2</td>
<td>0.821</td>
<td>0.096</td>
<td>0.356</td>
<td>0.744</td>
<td>0.039</td>
</tr>
<tr>
<td>0.3</td>
<td>0.735</td>
<td>0.168</td>
<td>0.399</td>
<td>0.666</td>
<td>0.068</td>
</tr>
<tr>
<td>0.4</td>
<td>0.633</td>
<td>0.251</td>
<td>0.442</td>
<td>0.573</td>
<td>0.101</td>
</tr>
<tr>
<td>0.5</td>
<td>0.552</td>
<td>0.368</td>
<td>0.485</td>
<td>0.473</td>
<td>0.140</td>
</tr>
<tr>
<td>0.6</td>
<td>0.459</td>
<td>0.442</td>
<td>0.528</td>
<td>0.416</td>
<td>0.178</td>
</tr>
<tr>
<td>0.7</td>
<td>0.356</td>
<td>0.585</td>
<td>0.571</td>
<td>0.323</td>
<td>0.235</td>
</tr>
<tr>
<td>0.8</td>
<td>0.256</td>
<td>0.702</td>
<td>0.614</td>
<td>0.232</td>
<td>0.282</td>
</tr>
<tr>
<td>0.9</td>
<td>0.135</td>
<td>0.833</td>
<td>0.657</td>
<td>0.122</td>
<td>0.335</td>
</tr>
</tbody>
</table>

It should be noted that the proposed normalization procedure for water-oil systems as outlined above could be extended to other systems, i.e., gas-oil or gas-water.

**THREE-PHASE RELATIVE PERMEABILITY**

The relative permeability to a fluid is defined as the ratio of effective permeability at a given saturation of that fluid to the absolute permeability at 100% saturation. Each porous system has unique relative permeability characteristics, which must be measured experimentally. Direct experimental determination of three-phase relative permeability properties is extremely difficult and involves rather complex techniques to determine the fluid saturation distribution along the length of the core. For this reason, the more easily measured two-phase relative permeability characteristics are experimentally determined.

In a three-phase system of this type it is found that the relative permeability to water depends only upon the water saturation. Since the water can flow only through the smallest interconnect pores that are present in the rock and able to accommodate its volume, it is hardly surprising that the flow of water does not depend upon the nature of the fluids occupying the other pores. Similarly, the gas relative permeability depends only upon the gas saturation. This fluid, like water, is restricted to a particular range of pore sizes and its flow is not influenced by the nature of the fluid or fluids that fill the remaining pores.
The pores available for flow of oil are those that, in size, are larger than pores passing only water, and smaller than pores passing only gas. The number of pores occupied by oil depends upon the particular size distribution of the pores in the rock in which the three phases coexist and upon the oil saturation itself.

In general, the relative permeability of each phase, i.e., water, gas, and oil, in a three-phase system is essentially related to the existing saturation by the following functions:

\[ k_{rw} = f(S_w) \]  
(5-41)

\[ k_{rg} = f(S_g) \]  
(5-42)

\[ k_{ro} = f(S_w, S_g) \]  
(5-43)

Function 5-43 is rarely known and, therefore, several practical approaches are proposed and based on estimating the three-phase relative permeability from two sets of two-phase data:

**Set 1: Oil-Water System**

\[ k_{row} = f(S_w) \]

\[ k_{rw} = f(S_w) \]

**Set 2: Oil-Gas System**

\[ k_{rog} = f(S_g) \]

\[ k_{rg} = f(S_g) \]

where \( k_{row} \) and \( k_{rog} \) are defined as the relative permeability to oil in the water-oil two-phase system and similarly \( k_{rog} \) is the relative permeability of oil in the gas-oil system. The symbol \( k_{ro} \) is reserved for the oil relative permeability in the three-phase system.

The triangular graph paper is commonly used to illustrate the changes in the relative permeability values when three phases are flowing simultaneously, as illustrated in Figures 5-10 and 5-11. The relative permeability data are plotted as lines of constant percentage relative permeability (oil, water, and gas isoperms). Figures 5-10 and 5-11 show that the relative permeability data, expressed as isoperms, are dependent on the saturation values for all three phases in the rock.
Three-Phase Relative Permeability Correlations

Honarpour, Keoderitz, and Harvey (1988) provided a comprehensive treatment of the two- and three-phase relative permeabilities. The authors listed numerous correlations for estimating relative permeabilities. The simplest approach to predict the relative permeability to the oil phase in a three-phase system is defined as:

\[ k_{ro} = k_{row} k_{rog} \]  

(5-44)

There are several practical and more accurate correlations that have developed over the years, including:

- Wyllie’s Correlations
- Stone’s Model I
- Stone’s Model II
- The Hustad-Holt Correlation
Wyllie’s Correlations

Wyllie (1961) proposed the following equations for three-phase relative permeabilities in a water-wet system:

**In a cemented sandstone, Vugular rock, or oolitic limestone:**

\[
k_{rg} = \frac{S_g^2 [ (1-S_{wc})^2 - (S_w + S_o - S_{wc})^2 ]}{(1-S_{wc})^4} \quad (5 - 45)
\]

\[
k_{ro} = \frac{S_o^3 (2S_w + S_o - 2S_{wc})}{(1 - S_{wc})^4} \quad (5 - 46)
\]
Stone's Model I

Stone (1970) developed a probability model to estimate three-phase relative permeability data from the laboratory-measured two-phase data. The model combines the channel flow theory in porous media with probability concepts to obtain a simple result for determining the relative permeability to oil in the presence of water and gas flow. The model accounts for hysteresis effects when water and gas saturations are changing in the same direction of the two sets of data.

The use of the channel flow theory implies that water-relative permeability and water-oil capillary pressure in the three-phase system are functions of water saturation alone, irrespective of the relative saturations of oil and gas. Moreover, they are the same function in the three-phase system as in the two-phase water-oil system. Similarly, the gas-phase relative permeability and gas-oil capillary pressure are the same functions of gas saturation in the three-phase system as in the two-phase gas-oil system.

Stone suggested that a nonzero residual oil saturation, called minimum oil saturation, $S_{om}$, exists when oil is displaced simultaneously by water and gas. It should be noted that this minimum oil saturation $S_{om}$ is different than the critical oil saturation in the oil-water system (i.e., $S_{orw}$) and the residual oil saturation in the gas-oil system, i.e., $S_{org}$. Stone introduced the following normalized saturations:

\[
\begin{align*}
    k_{rw} &= \left( \frac{S_w - S_{wc}}{1 - S_{wc}} \right)^4 \\
    k_{ro} &= \frac{(S_o)^3}{(1 - S_{wc})^3} \\
    k_{rg} &= \frac{(S_o)^3 (2S_w + S_o - 2S_{wc})^4}{(1 - S_{wi})^4}
\end{align*}
\]

In unconsolidated, well-sorted sand:

\[
\begin{align*}
    k_{rw} &= \left( \frac{S_w - S_{wc}}{1 - S_{wi}} \right)^3 \\
    k_{ro} &= \frac{(S_o)^3}{(1 - S_{wi})^3} \\
    k_{rg} &= \frac{(S_o)^3 (2S_w + S_o - 2S_{wc})^4}{(1 - S_{wi})^4}
\end{align*}
\]
The oil-relative permeability in a three-phase system is then defined as:

$$k_{ro} = S^*_o b_w b_g$$ (5-54)

The two multipliers $b_w$ and $b_g$ are determined from:

$$S^*_o = \frac{S_o - S_{om}}{(1-S_{wc}-S_{om})}, \text{for } S_o \geq S_{om}$$ (5-51)

$$S^*_w = \frac{S_w - S_{wc}}{(1-S_{wc}-S_{om})}, \text{for } S_w \geq S_{wc}$$ (5-52)

$$S^*_g = \frac{S_g}{(1-S_{wc}-S_{om})}$$ (5-53)

The oil-relative permeability in a three-phase system is then defined as:

$$k_{ro} = S^*_o b_w b_g$$ (5-54)

The two multipliers $b_w$ and $b_g$ are determined from:

$$b_w = \frac{k_{row}}{1-S^*_w}$$ (5-55)

$$b_g = \frac{k_{rog}}{1-S^*_g}$$ (5-56)

where $S_{om}$ = minimum oil saturation

$k_{row}$ = oil relative permeability as determined from the oil-water two-phase relative permeability at $S_w$

$k_{rog}$ = oil relative permeability as determined from the gas-oil two-phase relative permeability at $S_g$

The difficulty in using Stone’s first model is selecting the minimum oil saturation $S_{om}$. Fayers and Mathews (1984) suggested an expression for determining $S_{om}$.

$$S_{om} = \alpha S_{orw} + (1 - \alpha) S_{org}$$ (5-57)

with

$$\alpha = 1 - \frac{S_g}{1-S_{wc} - S_{org}}$$ (5-58)
where $S_{orw}$ = residual oil saturation in the oil-water relative permeability system

$S_{org}$ = residual oil saturation in the gas-oil relative permeability system

Aziz and Sattari (1979) pointed out that Stone’s correlation could give $k_{ro}$ values greater than unity. The authors suggested the following normalized form of Stone’s model:

$$k_{ro} = \frac{S_{o}^{*}}{(1-S_{ow}^{*})(1-S_{g}^{*})} \left( \frac{k_{row} k_{rog}}{(k_{ro})_{Swc}} \right)$$  \hspace{1cm} (5 - 59)$$

where $(k_{ro})_{Swc}$ is the value of the relative permeability of the oil at the connate water saturation as determined from the oil-water relative permeability system. It should be noted that it is usually assumed that $k_{rg}$ and $k_{rog}$ curves are measured in the presence of connate water.

**Stone’s Model II**

It was the difficulties in choosing $S_{om}$ that led to the development of Stone’s Model II. Stone (1973) proposed the following normalized expression:

$$k_{ro} = (k_{ro})_{Swc} \left[ \frac{k_{row}}{(k_{ro})_{Swc}} + k_{rw} \right]$$

$$\left[ \frac{k_{rog}}{(k_{ro})_{Swc}} + k_{rg} \right] - \left( k_{rw} + k_{rg} \right)$$ \hspace{1cm} (5 - 60)$$

This model gives a reasonable approximation to the three-phase relative permeability.

**The Hustad-Holt Correlation**

Hustad and Holt (1992) modified Stone’s Model I by introducing an exponent term $n$ to the normalized saturations to give:
where

\[ \beta = \frac{S_o^*}{(1 - S_w^*)(1 - S_g^*)} \]  

(5-62)

\[ S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om} - S_{gc}} \]  

(5-63)

\[ S_g^* = \frac{S_g - S_{gc}}{1 - S_{wc} - S_{om} - S_{gc}} \]  

(5-64)

\[ S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om} - S_{gc}} \]  

(5-65)

The \( \beta \) term may be interpreted as a variable that varies between zero and one for low- and high-oil saturations, respectively. If the exponent \( n \) is one, the correlation is identical to Stone’s first model. Increasing \( n \) above unity causes the oil isoperms at low oil saturations to spread from one another. \( n \) values below unity have the opposite effect.

**Example 5-5**

Two-phase relative permeability tests were conducted on core sample to generate the permeability data for oil-water and oil-gas systems. The following information is obtained from the test:

\[ S_{gc} = 0.10 \quad S_{wc} = 0.15 \]
\[ S_{orw} = 0.15 \quad S_{org} = 0.05 \]
\[ (k_{ro})_{S_{wc}} = 0.88 \]

At the existing saturation values of \( S_o = 40\% \), \( S_w = 30\% \), and \( S_g = 30\% \) the two-phase relative permeabilities are listed below:

\[ k_{row} = 0.403 \]
\[ k_{rw} = 0.030 \]
Estimate the three-phase relative permeability at the existing saturations by using:

a. Stone’s Model I
b. Stone’s Model II

**Solution**

**a. Stone’s Model I**

*Step 1.* Calculate $S_{om}$ by applying Equations 5-58 and 5-57, to give:

$$\alpha = 1 - \frac{0.3}{1 - 0.15 - 0.05} = 0.625$$

$$S_{om} = (0.625) (0.15) + (1 - 0.625)(0.05) = 0.1125$$

*Step 2.* Calculate the normalized saturations by applying Equations 5-51 through 5-53.

$$S_o^* = \frac{0.4 - 0.1125}{1 - 0.15 - 0.1125} = 0.3898$$

$$S_w^* = \frac{0.30 - 0.15}{1 - 0.15 - 0.1125} = 0.2034$$

$$S_g^* = \frac{0.3}{1 - 0.15 - 0.1125} = 0.4068$$

*Step 3.* Estimate $k_{ro}$ by using Equation 5-59.

$$k_{ro} = \frac{0.3898}{(1 - 0.2034) (1 - 0.4068) \left[ \frac{(0.406)(0.175)}{0.88} \right]} = 0.067$$

**b. Stone’s Model II**

Apply Equation 5-60 to give:
PROBLEMS

1. Given:
   • $S_{wc} = 0.30$  $S_{gc} = 0.06$  $S_{oc} = 0.35$
   • unconsolidated-well sorted sand

   Generate the drainage relative permeability data by using:
   a. The Wyllie-Gardner correlation
   b. Pirson’s correlation
   c. Corey’s method

2. The capillary pressure data for an oil-water system are given below:

<table>
<thead>
<tr>
<th>$S_w$</th>
<th>$p_c$ psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>35</td>
</tr>
<tr>
<td>0.30</td>
<td>16</td>
</tr>
<tr>
<td>0.40</td>
<td>8.5</td>
</tr>
<tr>
<td>0.50</td>
<td>5</td>
</tr>
<tr>
<td>1.00</td>
<td>0</td>
</tr>
</tbody>
</table>

   a. Generate the relative permeability data for this system.
   b. Using the relative permeability ratio concept, plot $k_{ro}/k_{rw}$ versus $S_w$ on a semi-log scale and determine the coefficients of the following expression:

   $$\frac{k_{ro}}{k_{rw}} = a e^{b S_w}$$

3. Using the relative permeability data of Example 5-6, generate the relative permeability values for a layer in the reservoir that is characterized by the following critical saturations:

   $S_{oc} = 0.25$  $S_{wc} = 0.25$  $h = 1$

4. Prepare a $k_{rg}/k_{ro}$ versus $S_g$ plot for the following laboratory data:

<table>
<thead>
<tr>
<th>$k_{rg}/k_{ro}$</th>
<th>$S_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>0.50</td>
</tr>
<tr>
<td>0.109</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Find the coefficients of the following relationship:

\[ \frac{k_{rg}}{k_{ro}} = ae^{bS_g} \]

REFERENCES