Gas formation volume factor.

The gas formation volume factor is used to relate the volume of gas, as measured at reservoir conditions, to the volume of the gas as measured at standard conditions, i.e., 60°F and 14.7 psia. This gas property is then defined as the actual volume occupied by a certain amount of gas at a specified pressure and temperature, divided by the volume occupied by the same amount of gas at standard conditions. In an equation form, the relationship is expressed as:

\[ \beta_g = \frac{\text{Vol. at reservoir condition}}{\text{Vol. at standard condition}} = \frac{V_r}{V_{sc}} \text{ in } \frac{\text{ft}^3}{\text{SCF}} \]

So \( \beta_g \) for the above example = 11.5/1000 = 0.0115 ft\(^3\)/SCF.

\[ \beta_g = \frac{V_r}{V_{sc}} = \frac{(\frac{znRT}{P})_{\text{res.}}}{(\frac{znRT}{P})_{\text{sc}}} = \frac{P_{sc}zT_r}{T_{sc}P_r} \]

Assuming that the standard conditions are represented by \( P_{sc} = 14.7 \text{ psia} \) and \( T_{sc} = 520 \), the above expression can be reduced to the following relationship:

\[ \therefore \beta_g = 0.028 \frac{zT_r}{P_r} \]

Calculate \( \beta_g \) for the previous example?

\[ \beta_g = 0.028 \frac{zT_r}{P_r} = 0.028 \frac{0.722 \times (460 + 104)}{1000} = 0.0114 \frac{\text{ft}^3}{\text{SCF}} \]

In other field units, the gas formation volume factor can be expressed in bbl/scf, to give:

\[ \beta_g = 0.005 \frac{zT_r}{P_r} \]
The reciprocal of the gas formation volume factor is called the gas expansion factor and is designated by the symbol \((E_g)\):

\[
E_g = 35.37 \frac{P}{zT} \text{ in } \frac{SCF}{ft^3} \quad \text{or} \quad E_g = 198.6 \frac{P}{zT} \text{ in } \frac{SCF}{bbl}
\]

**Example:** Calculate the gas formation volume factor and gas expansion factor of 500 SCF of gas at 1500 psi and \(T = 140^\circ C\). assuming the following composition:

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole fraction ((Y_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{CH}_4)</td>
<td>0.6</td>
</tr>
<tr>
<td>(\text{C}_2\text{H}_6)</td>
<td>0.3</td>
</tr>
<tr>
<td>(\text{C}_3\text{H}_8)</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Solution:** First of all we have to find \(Z_i\) for each component at given \(P\) and \(T\) from charts which:

\[
\begin{align*}
Z_i \text{ CH}_4 &= 0.99 \\
Z_i \text{ C}_2\text{H}_6 &= 0.75 \\
Z_i \text{ C}_3\text{H}_8 &= 0.435
\end{align*}
\]

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole fraction ((Y_i))</th>
<th>(Z_i) (from charts)</th>
<th>(Y_i Z_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{CH}_4)</td>
<td>0.6</td>
<td>0.99</td>
<td>0.594</td>
</tr>
<tr>
<td>(\text{C}_2\text{H}_6)</td>
<td>0.3</td>
<td>0.75</td>
<td>0.225</td>
</tr>
<tr>
<td>(\text{C}_3\text{H}_8)</td>
<td>0.1</td>
<td>0.435</td>
<td>0.0435</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Z_m = 0.8625)</td>
<td></td>
</tr>
</tbody>
</table>

\(T = 140^\circ C = 5/9 (^\circ F - 32) \Rightarrow T = 284^\circ F\)

\[
V_r = \frac{Z_r}{P_r T_r P_{s,c} T_{s,c}} = \frac{0.8625 \times (460 + 284) \times 14.7 \times 500}{1500 \times (460 + 60)}
\]
\[ \therefore V_r = 6.047 \, ft^3 \]

\[ \beta_g = \frac{V_r}{V_{sc}} = \frac{6.047}{500} = 0.012 \, \frac{ft^3}{SCF} \]

Or:

\[ \beta_g = 0.028 \frac{z \, T_r}{P_r} = 0.028 \frac{0.8625 \times 744}{1500} = 0.012 \, \frac{ft^3}{SCF} \]

\[ E_g = \frac{1}{\beta_g} = \frac{1}{0.012} = 83 \, \frac{SCF}{ft^3} \]

Or:

\[ E_g = 35.37 \frac{P}{zT} = 35.37 \frac{1500}{0.8625 \times 744} = 83 \, \frac{SCF}{ft^3} \]

**Pseudo critical and pseudo reduce properties.**

For single component system, the critical point is the highest value of pressure and temperature at which two phase can coexist. The pseudo critical properties can be obtained by using the partial volumes for mixtures to the critical properties of the mixtures gases.

The pseudo critical pressure and pseudo critical temperature are defined mathematically:

\[ pPc = \sum_{i=1}^{n} y_i P c_i \]

\[ pTc = \sum_{i=1}^{n} y_i T c_i \]
Where:

\( p_{PC} \) = Pseudo critical pressure

\( p_{TC} \) = Pseudo critical temperature

\( P_{ci} \) and \( T_{ci} \) = Critical pressure and temperature respectively of \( ith \) component

In cases where the composition of a natural gas is not available, the pseudo-critical properties, i.e., \( p_{PC} \) and \( p_{TC} \), can be predicted solely from the specific gravity of the gas. A graphical method for a convenient approximation of the pseudo-critical pressure and pseudo-critical temperature of gases has been presented when only the specific gravity of the gas is available. The correlation is presented in Figure below. This graphical correlation can be expressed in the following mathematical forms:

**For miscellaneous gases (natural gases) systems:**

\[
p_{TC} = 168 + 325 \, \gamma_g - 12.5 \, \gamma_g^2
\]

\[
p_{PC} = 677 + 15 \, \gamma_g - 37.5 \, \gamma_g^2
\]

**For gas condensate systems:**

\[
p_{TC} = 187 + 330 \, \gamma_g - 71.5 \, \gamma_g^2
\]

\[
p_{PC} = 706 - 51.7 \, \gamma_g - 11.1 \, \gamma_g^2
\]
Example:

Calculate pseudo critical pressure and pseudo critical temperature for the given gas mixture composition.(assume miscellaneous system).

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole fraction</th>
<th>P_c (psi)</th>
<th>T_c (°R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH₄</td>
<td>0.7</td>
<td>673</td>
<td>343</td>
</tr>
<tr>
<td>C₂H₆</td>
<td>0.2</td>
<td>708</td>
<td>550</td>
</tr>
<tr>
<td>C₃H₈</td>
<td>0.1</td>
<td>617</td>
<td>666</td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comp.</td>
<td>$Y_i$</td>
<td>$P_c$ (psi)</td>
<td>$T_c$ ($^\circ$R)</td>
<td>$M_W$</td>
<td>$M_{wi}$</td>
<td>= (2) * (5)</td>
<td>= (2) * (3)</td>
<td>= (2) * (4)</td>
</tr>
<tr>
<td>CH$_4$</td>
<td>0.7</td>
<td>673</td>
<td>343</td>
<td>16</td>
<td>11.2</td>
<td>471.1</td>
<td>240.1</td>
<td></td>
</tr>
<tr>
<td>C$_2$H$_6$</td>
<td>0.2</td>
<td>708</td>
<td>550</td>
<td>30</td>
<td>6</td>
<td>141.6</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>C$_3$H$_8$</td>
<td>0.1</td>
<td>617</td>
<td>666</td>
<td>44</td>
<td>4.4</td>
<td>61.7</td>
<td>66.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(M$_W$)$_m$ = 21.6</td>
<td>pP$_c$ = 674.4</td>
<td>pT$_c$ = 416.7</td>
</tr>
</tbody>
</table>

Or:

$$\gamma_g = \frac{M_a}{28.96} \approx \frac{21.6}{28.96} = 0.75$$

We can determine pP$_c$ and pT$_c$ either through the use of equations or using chart.

**Pseudo reduced properties**

The pseudo reduced properties are defined as the ratio of the property of the mixture to the pseudo critical property of the mixture. Thus the pseudo reduced pressure and pseudo reduced temperature are defined as:

$$ppr = \frac{P}{ppc}$$

$$ptr = \frac{T}{ptc}$$
Then by using $pPr$ and $pTr$, the value of Z-factor can be find by using Standing and Katz chart.

**Example:**

Calculate the density of the mixture in the previous example at 1000 psi and 104 °F.

**Solution:**

From previous example:

$M_a = 21.6$

$pP_c = 674.4$ psi psi

$pT_c = 416.7$ °R

$$pPr = \frac{P}{pP_c} = \frac{1000}{674.4} = 1.483 \quad , \quad pTr = \frac{T}{pT_c} = \frac{564}{416.7} = 1.353$$

From chart: $z = 0.798$

$$\rho = \frac{MP}{zRT} = \frac{21.6 \times 1000}{0.798 \times 10.73 \times 564} = 4.47 \text{ lb/ft}^3$$